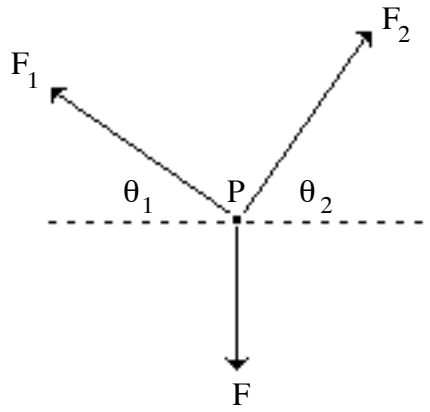
FINAL EXAM, Math 110, May 8, 1996 NAME _____

1. Consider the function $f(x) = 3x^{\frac{1}{3}}(x-8)$. What is the minimum value of this function?

   (a) 0 at $x = 0$ (b) $-18\sqrt[3]{2}$ at $x = 2$ (c) $-15\sqrt[3]{3}$ at $x = 3$

   (d) -27 at $x = -1$ (e) $-22\sqrt[3]{2}$ at $x = 2$

2. The three forces below are in equilibrium. They all act on the point P. The dotted line is horizontal and F acts vertically. If F_1 has a magnitude of 10 pounds, and if the angles θ_1 and θ_2 are 30^o and 60^o respectively, determine the magnitudes of F_2 and F.



3. Solve $5^{(x^2-1)} = 2$ for x.

   (a) $x = \sqrt{(\ln 5)/(\ln 2)}$ (b) $x = \sqrt{2}$ (c) $x = \pm\sqrt{\ln 2}$

   (d) $x = \sqrt{\ln 5}$ (e) $x = \pm\sqrt{(\ln 2)/(\ln 5)}$

4. Solve $\log_5(4x^2 - 11) = 2$ for x.

   (a) $x = \pm 2$ (b) $x = 3$ (c) $x = \pm 3$

   (d) $x = 4^{\ln 5}$ (e) $x = 0$

5. The derivatives of the functions $g(x) = \ln(x^4 + 3x^2)$ and $g(x) = e^{(x^2-1)}$ are respectively

   (a) $\frac{4x^3}{x^4 + 3x^2}$ and $2xe^{x^2-1}$ (b) $\frac{1}{x^4 + 3x^2}$ and e^{x^2-1}

   (c) $\frac{4x^3 + 6x}{x^4 + 3x^2}$ and $2xe^{x^2-1}$ (d) $\frac{1}{x^4 + 3x^2}$ and $2xe^{x^2-1}$

6. One of the waste products of a nuclear explosion is the radio-active isotope strontium–90. This isotope which behavers chemically like calcium, has a half-life of (about) 25 years. If 20 milligrams of the isotope are present in a sample now, in how many years will only 5 milligrams remain ?

(a) 35 years (b) 61 years (c) 49.5 years

(d) 53 years (e) 42.6 years

7. A radiation counter shows that a certain radioactive substance disintegrates at a rate of 5.2×10^{15} atoms per second at a certain time and at a rate of 3.7×10^{12} atoms 5 minutes later. The half-life of this radioactive substance is

(a) 21.02 seconds (b) 27 seconds (c) 41.6 seconds

(d) 34.65 seconds (e) 31.73 seconds

8. A fragment of a mineral grain is found to contain 305 parts per million rubidium–87 and 4.67 parts per million strontium–87. Given that the half-life of rubidium–87 is 4.7×10^{10} years, determine the age of the fragment. It is

(a) 1×10^9 years (b) 5×10^8 years (c) 3.6×10^8 years

(d) 1.5×10^9 years (e) 2.4×10^8 years

9. Fragments of skeletons unearthed near the town of Arella, Pennsylvania gave evidence of a civilization that existed in this area from around 14,300 to 15,000 years ago. Which of the numbers below is the best estimate of the ratio of stable carbon atoms to carbon–14 atoms in the fragments at the time of its discovery.

(a) 3.2×10^{11} (b) 6.1×10^{11} (c) 5.3×10^{12}

(d) 8×10^{12} (e) 3.8×10^{12}

10. An amount of \$5,000 is invested in an account that pays an interest rate of $r = 0.08$. What will the investment be worth after 7 years, if interest is compounded quarterly ?

(a) \$8,658.38 (b) \$8,705.12 (c) \$8,623.23

(d) \$8,831.50 (e) \$8,901.73

11. Your aunt has been paying \$20 per month into an account earning interest at an annual rate of 0.06 starting on the day you were born. She will give you all the money that has accumulated in this account on your 21st birthday. How much will you get?

(a) \$10,108 (b) \$6,032 (c) \$11,538

(d) \$9,560 (e) \$12,236

12. The CPI was 153.2 in September of 1995. Estimate the CPI in September of the year 2015 if the annual inflation rate over the twenty years in question will average 4% per year. Given that a Notre Dame education costs about \$25,000 per year in the academic year 1995/96, estimate the yearly cost of a Notre Dame education in the academic year 2015/2016.

(a) \$40,536 (b) \$100,500 (c) \$55,638

(d) \$50,783 (e) \$60,830

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1)	(a) ☐	(b) ☐	(c) ☐	(d) ☐	(e) ☐
2)	(a) ☐	(b) ☐	(c) ☐	(d) ☐	(e) ☐
3)	(a) ☐	(b) ☐	(c) ☐	(d) ☐	(e) ☐
4)	(a) ☐	(b) ☐	(c) ☐	(d) ☐	(e) ☐
5)	(a) ☐	(b) ☐	(c) ☐	(d) ☐	
6)	(a) ☐	(b) ☐	(c) ☐	(d) ☐	(e) ☐
7)	(a) ☐	(b) ☐	(c) ☐	(d) ☐	(e) ☐
8)	(a) ☐	(b) ☐	(c) ☐	(d) ☐	(e) ☐
9)	(a) ☐	(b) ☐	(c) ☐	(d) ☐	(e) ☐
10)	(a) ☐	(b) ☐	(c) ☐	(d) ☐	(e) ☐
11)	(a) ☐	(b) ☐	(c) ☐	(d) ☐	(e) ☐
12)	(a) ☐	(b) ☐	(c) ☐	(d) ☐	(e) ☐

A. The Williamsburg Bridge spans the East River in New York City. It has four cables. Its center span is 2d = 1600 feet, the dead load for its two decks is 19,210 pounds per foot and the live load capacity is 7,160 pounds. The sag s in the cable is 177 feet.

i. Compute the tension T_d in <u>one</u> of its cables at the tower, and the angle α that the cable makes (at the tower) with the horizontal.

$T_d =$; $\alpha =$

The Williamsburg Bridge is the only one of the East River suspension bridges for which the cables of the side span do not bear any of the load of the side span. The only purpose of each of these cables is to counterbalance T_d.

ii. If the cable over the side span makes an angle at the tower of 22.7^o with the horizontal compute the tension T in <u>one</u> of the cables of the side span.

T =

B. A wood fragment tested with an accelerator mass spectrometer and is found to contain carbon-14 atoms to stable carbon atoms in a ratio of 1 to 1.596×10^{12}. Compute the age of the fragment. Assume that at the time the metabolic processes in the wood stopped, the equilibrium ratio of radioactive carbon to stable carbon was the same as it is today.

years

C. A professor turned 30 in May of 1994 and has (since that time) been paying $1,000 per month into an account that pays at an annual rate of $r = 0.04$ compounded monthly. How much money will she have in this account when she retires at the age of 65 in May of the year 2029 ? She has decided to use this account to provide her with monthly annuity payments for the first 10 years of her retirement. What monthly payments will she receive from this account if it pays interest at an annual rate of $r = 0.05$ compounded monthly?

Ans:

D. The short run supply and demand functions for a given product in a given market are both linear. At a price of 7 dollars, the demand and supply are both 160,000 units per month. At this price the price elasticity of supply is 0.20 and the price elasticity of demand is –0.12.

i. Determine the supply function for the product.

S(p) =

ii. Determine the demand function for the product.

D(p) =

iii. A cartel of consumers get together and are able to cut the demand by 20,000 units. Assuming that the short run supply situation remains the same, estimate the new equilibrium price that market forces will determine.

Ans:

E. A firm's marginal cost function is given by $MC(x) = 0.0001\ x^2 + 0.4x + 500$, where x is the number of units produced per year. Assume that the firm operates in a stable and competitive market and that the firm sells everything it produces. Estimate the firm's profit maximizing output if the market price is $120 per unit.

(a) $F_2 = 10\sqrt{3}$; $F = 20$

(b) $F_2 = 10$; $F = 20\sqrt{3}$

(c) $F_2 = 10\sqrt{3}$; $F = 15$

(d) $F_2 = 20$; $F = 10\sqrt{3}$

(e) $F_2 = 10$; $F = 20$