FINAL Math 110, May 6, 1998 NAME ________________________
ID ____________________

PLEASE NOTE: PLACE YOUR ANSWER IN THE BOX PROVIDED. FOR FULL CREDIT, SHOW THE DETAILS OF YOUR WORK IN A NEAT AND ORGANIZED WAY IN THE SPACE PROVIDED. REMEMBER THE STRICT OBSERVANCE OF THE HONOR CODE.

1. Consider the function $f(x) = (x^2 - 5x)^{\frac{1}{3}}$.

(i) For which values of x does its graph cross the x-axis?

x = ,

(ii) Compute the derivative of $f(x)$.

f'(x) =

(iii) Find all the critical numbers of f(x).

Critical numbers:

(iv) Determine precisely the portions of the x-axis for which the function $f(x)$ is increasing and decreasing.

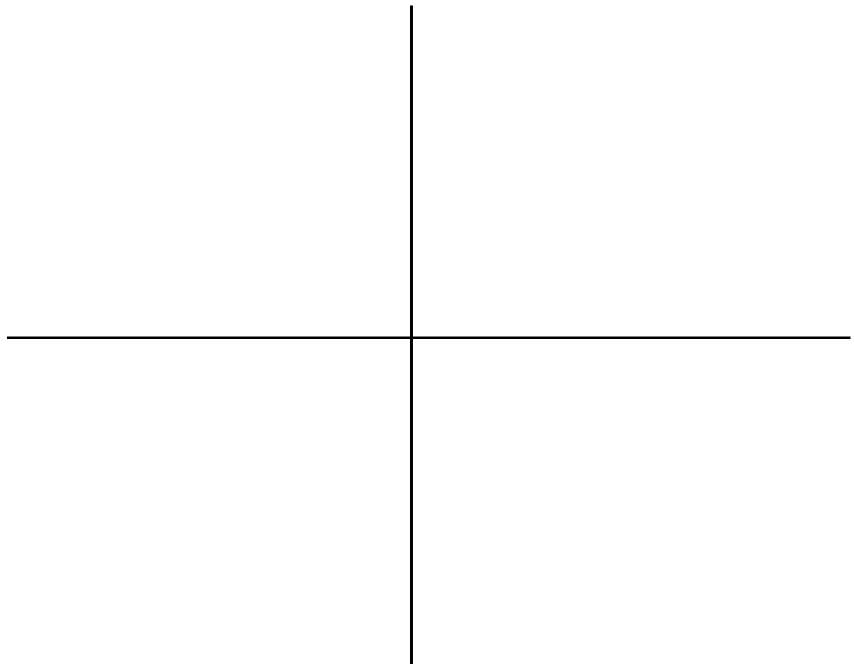
Increasing:	Decreasing:

(v) At which values of x does $f(x)$ have local maxima; at which does it have local minima?

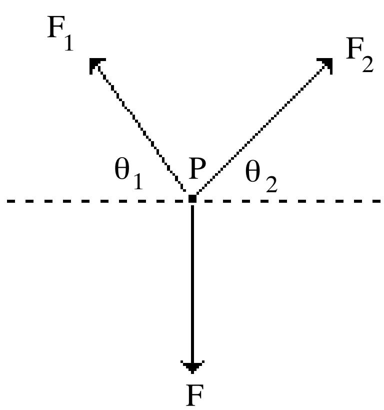
Local Maxes:	Local Mins:

(vi) What can you say about the value of $f(x)$ when x is large and positive? When x is large and negative?

(vii) Sketch the graph of f based on the information that you have developed in points (i) – (vi) (do not go to the effort of computing the second derivative). Plot the important points.



2. The three forces below are in equilibrium. They all act on the point P. The dotted line is horizontal and F acts vertically. If F_1 has a magnitude of 30 pounds, and if the angles θ_1 and θ_2 are 70^o and 50^o respectively, determine the magnitudes of F_2 and F.



F = , F_2 =

3. The Tacoma Narrows Bridge was completely rebuilt in the years 1948 to 1950. The data for the new bridge is as follows. It has a total length of 5,000 feet and a center span of 2,800 feet. Its 2 main cables support a single deck that carries 4 lanes of automobile

traffic. The dead load is 8,680 pounds per foot. Assume that it is designed for a live load capacity of 4,000 pounds per foot and that the sag in the cable over the center span is 280 feet.

(i) Consider one of the main cables over the center span at a point where it meets one of the towers. Compute the tension T of the cable at that point and compute the angle α that the cable makes with the horizontal at that point.

$T =$, $\alpha =$

(ii) Compute the compression C that <u>both</u> of these cables generate in <u>one</u> tower.

$C =$

4. Find all the values of x that satisfy $\ln (x + 6) + \ln (x - 3) = \ln 5 + \ln 2$.

$x =$

5. Compute the following derivatives

a) $f(x) = \ln x$

b) $g(x) = \left[\ln (x^4 + 5x^2)\right]^3$

$f'(x) =$ ________________

$g'(x) =$ ____________________________________

c) $f(x) = e^x$

d) $g(x) = xe^{(x^2 - 1)}$

$f'(x) =$ ________________

$g'(x) =$ ______________________________

6. Rutherford studied the radium isotope $^{224}_{88}$Ra. In one experiment he made the following measurements for the decay rate $y'(t)$ of the sample he was testing:
at $t = 1$ day, $\frac{y'(t)}{y'(0)} = 0.88$; at $t = 2$ days, $\frac{y'(t)}{y'(0)} = 0.72$; at $t = 4$ days, $\frac{y'(t)}{y'(0)} = 0.53$; at $t = 7$ days, $\frac{y'(t)}{y'(0)} = 0.295$; at $t = 8$ days, $\frac{y'(t)}{y'(0)} = 0.252$; at $t = 11$ days, $\frac{y'(t)}{y'(0)} =$ 0.152; at $t = 13$ days, $\frac{y'(t)}{y'(0)} = 0.111$.

(i) How did Rutherford deduce the value of the disintegration constant λ of radium–224 could from this data and what is this value?

$\lambda =$

(ii) Suppose that Rutherford's sample consists of 15 milligrams at time $t = 0$. How many milligrams S will there be in the sample after precisely 5 days 6 hours?

$S =$

7. A wood fragment tested with an accelerator mass spectrometer and is found to contain carbon-14 atoms to stable carbon atoms in a ratio of 1 to $1.327 \infty 10^{12}$. Compute the age of the fragment. Assume that at the time the metabolic processes in the wood stopped, the equilibrium ratio of radioactive carbon to stable carbon was 93% of what it is today.

years

8. A fragment of a grain of volcanic ash is tested in a laboratory and found to be between 2 million and 2.2 million years old. Estimate the ratio R of argon–40 atoms to potassium–40 atoms in the fragment tested. State your answer in terms of two inequalities.

____________________ $\leq$ R $\leq$ ____________________

9. Suppose that a culture of bacteria is in its exponential phase of growth and contains 300,000 bacteria at a certain time. How many bacteria will there be in the culture 2 hours later if the doubling time is 43 minutes.

Ans:

10. Evaluate $\int \frac{2x - 6}{(4 - x)(x + 2)}\, dx$

Ans:

11. The statistics for the population of a certain country show the following:

8.45 million at the beginning of 1975,
and an increase at a rate of 0.21 million per year in 1975;
9.56 million at the beginning of 1980,
and an increase at a rate of 0.23 million per year in 1980;
10.77 million at the beginning of 1985,
and an increase at a rate of 0.25 million per year in 1985;
12.09 million at the beginning of 1990,
and an increase at a rate of 0.27 million per year in 1990;
13.52 million at the beginning of 1995,
and an increase at a rate of 0.30 million per year in 1995.

Let $t = 0$ correspond to the year 1975. Let $y(t)$ be the population of this country at any time $t \geq 0$ in years.

(i) Verify that the population of this country satisfies the basic assumption of the logistics model by setting up an approriate table and sketching an appropriate graph. (Work with an accuracy to within 2 decimal places.)

(ii) What is the limit on the population of this country ?

Ans:

(iii) Insert all relevant constants and determine $y(t)$.

$y(t) =$

12. The data below can be extracted from the world's population statistics. For each year: y is the population count in billions at the beginning of that year and $\frac{y'}{y}$ is the specific growth rate. Consider the beginning of 1965 to be $t = 0$.

	1965	1970	1975	1980	1985	1990	1995
y	3.34	3.70	4.08	4.45	4.85	5.29	5.77
$\frac{y'}{y}$	0.021	0.020	0.017	0.018	0.018	0.017	0.016

Assume that the population satisfies Gompertz's formula $\frac{y'(t)}{y(t)} = mke^{-kt}$.

(i) Use this formula and the information in the table for 1965 and 1995 to determine m and k.

m = , k =

(ii) Make a table that compares for t = 5, 10, 15, 20, and 25 the actual value of $\frac{y'(t)}{y(t)}$ with that given by Gompertz's formula.

(iii) What is the limit (in billions) on the world's population that the Gompertz model determines?

Ans:

(iv) Use the model to predict the world's population in the year 2100.

Ans:

FORMULAS

$T_0 = T_x \cos\theta \qquad wx = T_x \sin\theta \qquad \frac{wx}{T_0} = \tan\theta \qquad f(x) = \frac{w}{2T_0} x^2$

$T_x = w\sqrt{\frac{1}{4}\frac{d^4}{s^2} + x^2} \qquad \tan\alpha = f'(d) = \frac{2s}{d^2} \quad d = \frac{2s}{d}$

$\frac{\ln\left(\frac{y'(t)}{y'(0)}\right)}{t} = -\lambda \qquad t = \frac{1}{\lambda}\ln\left(\frac{z(t)}{y(t)} + 1\right) \qquad t = (1.89 \infty 10^9)\ln\left(\frac{9.07z(t)}{y(t)} + 1\right)$

$y(t) = y_0 e^{-\lambda t} \qquad y'(t) = -\lambda y(t) \qquad 1\text{ g} = \frac{1}{m}(6.02 \infty 10^{23})$

$t = (8.26 \infty 10^3)\ln\left(r_0 \frac{k}{y(t)}\right) \qquad r_0 = \frac{1}{6.463 \infty 10^{11}}$

$y(t) = y_0 e^{-\lambda t} \qquad h = \frac{\ln 2}{\lambda} \qquad y(t) = y_0 e^{\mu t} \qquad d = \frac{\ln 2}{\mu}$

$M = \mu k^{-1} \qquad \frac{y'}{y} = \mu - ky$

$$y(t) = \frac{M e^{\mu t}}{\frac{M}{y_0} - 1 + e^{\mu t}} \quad \text{or} \quad y(t) = \frac{M}{1 + \left(\frac{M}{y_0} - 1\right) e^{-\mu t}}$$

$$\frac{y'(t)}{y(t)} = mke^{-kt} \qquad y(t) = y_0 e^{m - me^{-kt}}$$