MIDTERM, Math 110, March 3, 1998 NAME ____________________________________

Show all your work. If answers are provided, **Circle the correct one.**

1. Consider the function $f(x) = 3x^{\frac{1}{3}}(x-8)$. Determine the intervals over which the graph of this function is increasing; also determine the intervals over which it is decreasing.

Increasing: ______________________________ Decreasing: ______________________________

What is the minimum value of this function?

(a) 0 at $x = 0$ (b) $-18\sqrt[3]{2}$ at $x = 2$ (c) $-15\sqrt[3]{3}$ at $x = 3$

(d) -27 at $x = -1$ (e) $-22\sqrt[3]{2}$ at $x = 2$

2. The Williamsburg Bridge spans the East River in New York City. It has four cables. Its center span is $2d = 1600$ feet, the dead load for its two decks is 19,210 pounds per foot and the live load capacity is 7,160 pounds. The sag s in the cable is 177 feet.

i. Compute the tension T in one of its cables at the tower, and the angle α that the cable makes (at the tower) with the horizontal.

T = ; α =

The Williamsburg Bridge is the only one of the East River suspension bridges for which the cables of the side span do not bear any of the load of the side span. The only purpose of each of these cables is to counterbalance T.

ii. If the cable over the side span makes an angle at the tower of 22.7^{o} with the horizontal compute the tension T_s in <u>one</u> of the cables of the side span. Draw a diagram of what is going on.

T_s =

3. Differentiate $y = 3x^2e^{(x^3-x)}$ and $z = (\ln(1+t^2))^2$.

$\frac{dy}{dx}$ =

$\frac{dz}{dt}$ =

4. Solve $\ln(4x^2 - 11) = 2$ for x.

(a) $x = \pm 2$ (b) $x = 3$ (c) $x = \pm 3$ (d) $x = 4^{\ln 5}$ (e) $x = 0$

5. One of the waste products of a nuclear explosion is the radio-active isotope strontium–90. This isotope which behaves chemically like calcium, has a half-life of (about) 25 years. If 20 milligrams of the isotope are present in a sample now, in how many years will only 5 milligrams remain ?

(a) 35 years (b) 61 years (c) 50 years (d) 53 years (e) 43 years

6. A radiation counter shows that a certain radioactive substance disintegrates at a rate of $7.33 \infty 10^{14}$ atoms per second at a certain time and at a rate of $4.82 \infty 10^{12}$ atoms per second 7 minutes later. The half-life of this radioactive substance is

(a) 21.02 seconds (b) 27 seconds (c) 41.6 seconds
(d) 34.65 seconds (e) 31.73 seconds

USEFUL FORMULAS

$T_0 = T_x \cos\theta \qquad wx = T_x \sin\theta \qquad \frac{wx}{T_0} = \tan\theta \qquad f(x) = \frac{w}{2T_0} x^2$

$T_x = w\sqrt{\frac{1}{4}\frac{d^4}{s^2} + x^2} \qquad \tan\alpha = f'(d) = \frac{2s}{d^2} \quad d = \frac{2s}{d}$

$T_d = wd\sqrt{\left(\frac{d}{2s}\right)^2 + 1} \qquad \tan\alpha = f'(d) = \frac{2s}{d^2} \quad d = \frac{2s}{d}$

$y(t) = y_0 e^{-\lambda t} \qquad y'(t) = -\lambda y(t) \qquad 1 \text{ gram} = \frac{1}{m}(6.02 \infty 10^{23}) \text{ atoms} \qquad h = \frac{\ln 2}{\lambda}$

$\ln\left(\frac{y'(t)}{y'(0)}\right) = -\lambda t \qquad t = \frac{1}{\lambda}\ln\left(\frac{z(t)}{y(t)} + 1\right) \qquad t = (1.89 \infty 10^9)\ln\left(\frac{9.07z(t)}{y(t)} + 1\right)$

$1 \text{ milligram} = 10^{-3} \text{ grams} \qquad \log_a x = \frac{\log_b x}{\log_b a}$