date: the 16^{th} of October, 1998

place: room 221 Hayes

time: 8:30-9:20

111 - Exam II

This exam contains 10 problems worth 10 points each. You may use only a blank sheet of paper, a pencil, a rubber gum, a ruler and a small calculator. You can use your calculator only to add, substract, multiply or divide two numbers. This exam is taken under the honor code.

Name:

Recommendation

Never give a "solitary" answer without justifying it by previous calculations or reasoning.

Problems

1. Calculate

$$\lim_{x\to\infty} \frac{2x^3 - x^2 + x - 1}{x^3 - x + 1} \ .$$

Give a geometrical interpretation for this limit.

2. Calculate the first derivative of the function

$$f(x) = \frac{x^3 + 1}{x^2 + 1}$$

and bring it to the simplest form you can.

3. Calculate $f'(2)$ where $f(x) = (x^2 + x + 1)^2$. (*Hint: First, calculate* $f'(x)$.)

4. Calculate the second derivative of the function $f(t) = t^3 - t^2 + t - 1$.

5. Calculate the limit

$$\lim_{x\to -2} \frac{x^2 + 5x + 6}{2x + 4} \ .$$

6. Sketch the graph of a function $f(x)$ knowing that $f(1) = -1$, $f'(1) = 0$, $f''(1) = 1$, $f(-2) = 2$, $f'(-2) = 0$, $f''(-2) = -2$, $f(0) = 0$. Specify a minimum point and a maximum point for this function.

7. Sketch as accurately as possible the graph of the function $f(x) = -x^2 + 2x + 3$. Clearly plot any x-intercept, y-intercept, and extreme point.

8. Show that the function $f(x) = x^5 + x^3 + x$ is increasing for all x. (*Hint: Observe that* 1, x^2, *and* x^4 *are never negative quantities.*)

9. Find the inflection point of the function $f(x) = \frac{x^3}{3} + \frac{5x^2}{2} + 6x$ and state where the function is concave up and where concave down.

10. A rectangular box is made of three kinds of material. The top face costs \$ 5 per square foot, the bottom face \$ 10 per square foot, and each lateral face \$ 6 per square foot. Suppose that the bottom (top) face is a square and the volume of the box is 5 square feet. Is \$ 44 enough to construct such a box? Justify your answer. (*Hint: First, find the minimum cost of the box.)*

Good luck!