Exam. II

Math. 112, Spring 1997

Your name:

The room is too small to allow for alternate seating. Recall that you are on the honor system. Try to keep your eyes on your own work, and to place your work where it will not distract your neighbors.

The exam consists of 10 problems. There is some space for work and answers provided in the test booklet. If you use further sheets of paper, please label them with your name and the problem number(s).

The space below should be left blank--it will be used when your paper is being graded.

1.

2.

3.

4.

5.

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7.

8.

9.

10.

1. (a) Fill in the truth table for the formula $(\neg(P \rightarrow Q) \rightarrow P)$.

P	Q	$(P \rightarrow Q)$	$\neg(P \rightarrow Q)$	$(\neg(P \rightarrow Q) \rightarrow P)$
F	F			
F	T			
T	F			
T	T			

(b) Say whether this formula is tautologous, contingent, or inconsistent.

2. (a) Fill in the truth table for the formula $((P \vee (Q \,\&\, R)) \,\&\, \neg(P \vee R))$.

P	Q	R	$(Q \,\&\, R)$	$(P \vee (Q \,\&\, R))$	$(P \vee R)$	$\neg(P \vee R)$	$((P \vee (Q \,\&\, R)) \,\&\, \neg(P \vee R))$
F	F	F					
F	F	T					
F	T	F					
F	T	T					
T	F	F					
T	F	T					
T	T	F					
T	T	T					

(b) Say whether this formula is tautologous, contingent, or inconsistent.

3. Determine whether $(P \rightarrow Q), (Q \rightarrow \neg P) \models \neg Q$.

Justify your answer by either giving an assignment of truth values which makes the premises true and the conclusion false, or explaining why there is no such assignment.

4. Determine whether $(P \rightarrow ((Q \vee R) \rightarrow S)), (P \rightarrow ((\neg Q \vee \neg R) \rightarrow \neg S), (Q \ \& \ \neg R) \models \neg P$.

Justify your answer by either giving an assignment of truth values which makes the premises true and the conclusion false, or else explaining why there is no such assignment.

5. Finish the statement of the Completeness Theorem below.

<u>Theorem</u>: For any propositional statements $A_1,...,A_n$ and B,

if ____,

then ____.

6. Translate the following so as to make a <u>sound</u> argument in propositional logic.

John and Sarah plan to see "Shine", or "The English Patient", or "Star Wars".
Sarah refuses to see "Star Wars" again.
John has no interest in "The English Patient".
So, they must be going to see "Shine".

Let H--John sees "Shine", H'--Sarah sees "Shine", E--John sees the "English Patient", E'--Sarah sees the "English Patient", W--John sees "Star Wars", W'--Sarah sees "Star Wars".

<u>Hint</u>: Take (H & H') ∨ (E & E') ∨ (W & W') as your first premise.

7. Match the English sentences below with their translations in predicate logic.

Let C, O, and F be 1-place predicates standing, respectively, for being a Chemist, liking opera, and liking football.

(a) Some chemists like opera, and some chemists like football.

(b) Some chemists like both opera and football.

(c) No chemists like both opera and football.

(d) Chemists are the only ones who like both opera and football.

(i) $(\exists x)\ (Cx\ \&\ (Ox\ \&\ Fx))$

(ii) $(\exists x)\ (Cx\ \&\ Ox)\ \&\ (\exists x)\ (Cx\ \&\ Fx)$

(iii) $(x)\ ((Ox\ \&\ Fx) \rightarrow Cx)$

(iv) $\neg(\exists x)\ (Cx\ \&\ (Ox\ \&\ Fx))$

8. For each of the following predicate formulas, say whether it is a sentence, and if not, give the variables which occur free. Here E is a 2-place predicate, and m is a constant.

(a) Emm

(b) $(\exists y)\ (Exy\ \&\ \neg x = y)$

9. Let Bxyz be a 3-place predicate saying (of numbers x, y, and z) that y is between x and z.

Write a sentence saying that for any pair of <u>distinct</u> numbers, there is a number between the two.

10. Give a formation sequence showing that the following is a wff in predicate logic.

(x) (∃y) Pxy (here P is a 2-place predicate symbol).