

Exam. III

Math. 112, Spring 1997

Your name:

The room is too small to allow for alternate seating. Recall that you are on the honor system. Try to keep your eyes on your own work, and to place your work where it will not distract your neighbors.

The exam consists of 8 problems. Problem 7 is worth 16 points; the other problems are worth 12 points each. There is some space for work and answers provided in the test booklet. If you use further sheets of paper, please label them with your name and the problem number(s).

The space below should be left blank--it will be used when your paper is being graded.

- 1.
- 2.
- 3.
- 4.
- 5.
- 6.
- 7.
- 8.



On problems 1-4, give a proof using the primitive rules for predicate logic without equality; i.e., the 10 propositional rules and 4 quantifier rules.

1.  $(x) (Fx \rightarrow Gx), Fc \vdash Gc$

2.  $(x) (y) Hxy \vdash (y) (x) Hxy$

3.  $(\exists x) Fx, (x) \neg Gx \vdash (\exists x) (Fx \& \neg Gx)$

4.  $(Fa \vee Fb) \vdash (\exists x) Fx$

On problems 5-6, give a complete proof using the primitive rules for predicate logic with equality.

5.  $\vdash (x) x = x$

6.  $a = b \vdash (Lab \rightarrow Lbb)$

7. For each of the English sentences below, give the letter corresponding to the best translation. Let  $Lxy$  stand for  $x$  likes  $y$ . There is no predicate for people (people are the only objects referred to here).

- |                                    |                               |
|------------------------------------|-------------------------------|
| (i) Everyone likes someone.        | (a) $(\exists x)(y) \neg Lxy$ |
| (ii) Everyone is liked by someone. | (b) $(x)(\exists y) Lyx$      |
| (iii) Some people like noone.      | (c) $(\exists x)(y) Lxy$      |
| (iv) Some people like everyone.    | (d) $(x)(\exists y) Lxy$      |

8. In the following "proof", one line is not properly justified. Find the line and explain what is wrong.

- |   |                     |       |    |
|---|---------------------|-------|----|
| 1 | 1. $(\exists x) Fx$ |       | A  |
| 2 | 2. $Fa$             |       | A  |
| 2 | 3. $(x) Fx$         | 2     | UI |
| 1 | 4. $(x) Fx$         | 1,2,3 | EE |