Final Exam.

Math. 112, Spring 1997

Your name:

The exam consists of ? problems. There is some space for work and answers provided in the test booklet. If you use further sheets of paper, please label them with your name and the problem number(s).

The space below should be left blank--it will be used when your paper is being graded.

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1. Show, by giving a formation sequence, that $(\neg(P \& Q) \rightarrow R)$ is a wff of propositional logic.

2. Show that $(P \rightarrow Q)$, $(Q \rightarrow R) \vdash (P \rightarrow R)$. [Give a proof using only primitive rules.]

3. Show that (P → Q) |- (¬Q → ¬P). [Give a proof using only primitive rules.]

4. Show that (P v Q), (P → R), (Q → R) |- R. [Give a proof using only primitive rules.]

5. Complete the proof showing that ⊢ (P ∨ ¬P), by filling in appropriate decorations on the right.

1	1. ¬(P ∨ ¬P)
2	2. P
2	3. (P ∨ ¬P)
1,2	4. ((P ∨ ¬P) & ¬(P ∨ ¬P))
1	5. ¬P
1	6. (P ∨ ¬P)
1	7. ((P ∨ ¬P) & (P ∨ ¬P))
	8. ¬¬(P ∨ ¬P)
	9. (P ∨ ¬P)

6. (a) Complete the truth table for the formula (¬(P & Q) & P)

P	Q	(P & Q)	¬(P & Q)	(¬(P & Q) & P)
F	F			
F	T			
T	F			
T	T			

(b) Say whether the formula from part (a) is tautologous, contingent, or inconsistent.

7. Translate the following argument into propositional logic, letting G--suspect is guilty, J--suspect will go to jail, L--witness lied, M--moon is made of green cheese..

Premises: 1. Either the suspect is guilty or the witness lied.

               2. If the suspect is guilty, he will go to jail.

               3. If the witness lied, then the moon is made of green cheese.

               4. The moon is not made of green cheese.

Conclusion: The suspect will go to jail.

8. Determine whether the following argument is sound. Justify your answer by either giving an assignment of truth-values to the propositional variables which makes the premises all true and the conclusion false, or arguing that there is no such assignment.

Premises: 1. (P v Q)

               2. (P → R)

               3. (Q → S)

               4. ¬S

Conclusion: R

9. For each of the following formulas of predicate logic, say whether it is a sentence, and if not, give the free variables. [Here F is a 2-place predicate, and j and p are constants.]

(a) Fjp

(b) (y) (Fxy → Fyz)

10. Match the English sentences below with their translations into predicate logic, letting Fxy stand for x is a friend of y, and letting j and p stand for John and Pam, respectively.

(a) Pam is a friend of John.

(b) Some friends of Pam are friends of John.

(c) Any friend of John is a friend of Pam.

(d) John and Pam have no friends in common.

(i) ¬(∃x) (Fxj & Fxp)

(ii) (∃x) (Fxp & Fxj)

(iii) (x) (Fxj → Fxp)

(iv) Fpj

11. Complete the proof that ¬(∃x) Fx ⊢ (x) ¬Fx, by filling in appropriate decorations on the right.

1   1. ¬(∃x) Fx

2   2. Fc

2   3. (∃x) Fx

1,2   4. (∃x) Fx & ¬(∃x) Fx

1   5. ¬Fc

1   6. (x) ¬Fx

12. Complete the proof that a = b ⊢ b = a, by filling in appropriate decorations on the right.

1   1. a = b

        2. a = a

1   3. b = a

13. Give a proof that (x) (Gx → Hx), Gc |- Hc. [Use only primitive rules.]

14. Give a proof that (x) (Gx → Hx), (∃x) Gx |- (∃x) Hx. [Use only primitive rules.]

15. Give a proof that |- (x) x = x. [Use only primitive rules.]

16. Of the topics covered in the talks, which one, aside from your own, seems most interesting. Explain briefly (one or two sentences).