Solutions to Exam. I

1. It is not a Friday in June.
2. It is niether Friday nor June.
3. Letting P and Q be as before, and taking the translations from 1 and 2, ¬(P&Q) would be true on a Friday in February, while (¬P&¬Q) would not. Assuming Soundness, it follows that ¬(P & Q) |⁄ (¬P&¬Q) .

4. The definition of wff says (1) a propositional variable is a wff, (2) if F is a wff, so is ¬F, (3) if F and G are wff's, so are (F & G), (F ∨ G), and (F → G), and (4) nothing is a wff unless it can be obtained by finitely many applications of (1), (2), (3). The sequence below witnesses the fact that ¬(P ∨ Q) is a wff:

1. P, application of (1)
2. Q, application of (1)
3. (P ∨ Q), application of (3) to 1 and 2
4. ¬(P ∨ Q), application of (2) to 3

5.

1	1. P		A
2	2. Q		A
1,2	3. (P & Q)	1,2	&I
1	4. (Q → (P & Q))	2,3	CP
	5. (P → (Q → (P & Q)))	1,4	CP

6.

1	1. P		A
1	2. ¬¬P	1	DN
	3. (P → ¬¬P)	1,2	CP

7.

1	1. (P → Q)		A
2	2. (Q → R)		A
3	3. ¬R		A
2,3	4. ¬Q	2,3	MTT
1,2,3	5. ¬Q	1,4	MTT

8.

1	1. (P ∨ Q)		A
2	2. (P → Q)		A
3	3. P		A
2,3	4. Q	3,2	MPP
5	5. Q		A
1,2	6. Q	1,3,4,5,5	∨E

9.

2	3.	(Decorations on the left indicate assumptions being used.)
1,2	4.	
1	5.	

10.

5. 2,3		(Decorations on the right indicate earlier lines to which
6. 4,5		rule is being applied.)
7. 6		

9. 1,3,7,8,,8
10. 2,9