

### Solutions to more miscellaneous problems

1. Formation sequence for  $((P \& \neg Q) \vee R) \leftrightarrow ((R \& \neg Q) \rightarrow \neg P)$  (recall that  $\leftrightarrow$  is an abbreviation):  $P, Q, R, \neg Q, \neg P, (P \& \neg Q), ((P \& \neg Q) \vee R), (R \& \neg Q), ((R \& \neg Q) \rightarrow \neg P), (((P \& \neg Q) \vee R) \rightarrow ((R \& \neg Q) \rightarrow \neg P)), (((R \& \neg Q) \rightarrow \neg P) \rightarrow ((P \& \neg Q) \vee R)), (((P \& \neg Q) \vee R) \rightarrow ((R \& \neg Q) \rightarrow \neg P)) \& (((R \& \neg Q) \rightarrow \neg P) \rightarrow ((P \& \neg Q) \vee R))$

The formula is contingent. Suppose  $R$  is true and  $Q$  is false. Then if  $P$  is true, the formula is false, while if  $P$  is false the formula is true. (Some calculations are being suppressed here.)

2. (a)  $P, (P \rightarrow Q) \vdash (P \& Q)$
- (b)  $\neg P, (Q \rightarrow P) \vdash (\neg P \& \neg Q)$
- (c)  $\vdash (P \rightarrow (P \vee Q))$
- (d)  $(P \rightarrow Q), (Q \rightarrow R) \vdash (P \rightarrow R)$
- (e)  $P, (R \rightarrow \neg P) \vdash \neg R$
- (f)  $(R \rightarrow P), (R \rightarrow \neg P) \vdash \neg R$
- (g)  $(P \vee Q), (P \rightarrow R), (Q \rightarrow R) \vdash R$

3. (a) Classes are canceled if either it is snowing or the temperature is below 0. Let  $C$ --classes are canceled,  $S$ --it is snowing,  $T$ --the temperature is below 0. Then this is  $((S \vee T) \rightarrow C)$
- (b) It is Friday but the sun is not shining. Let  $F$ --it is Friday,  $S$ --the sun is shining. Then this is  $(F \& \neg S)$
- (c) Tom embezzled money from ABAG, and if he is caught he will go to jail. Let  $E$ --Tom embezzled,  $C$ --he is caught,  $J$ --he will go to jail. Then this is  $(E \& (C \rightarrow J))$

4. If the river floods, then our entire wheat crop will be destroyed. The river will flood if there is an early thaw. In any case, there will be heavy rains later in the summer. Therefore, if there is an early thaw, our entire community will be bankrupt unless there are heavy rains later in the summer.

Let  $F$ --river floods,  $D$ --wheat crop destroyed,  $E$ --early thaw,  $B$ --community bankrupt,  $R$ --heavy rains later. Then we have  $(F \rightarrow D), (E \rightarrow R), R \models (\neg R \rightarrow (E \rightarrow B))$ .

If even the one premise  $R$  is true, the conclusion is true, so the argument is sound.

### Predicate logic

1. Formation sequence for  $(\forall y) (Rxy \rightarrow Gx)$ :  $Rxy, Gx, (Rxy \rightarrow Gx), (\forall y) (Rxy \rightarrow Gx)$ .
2. (a)  $(\forall x) (\forall y) Fxy$  is a sentence
- (b)  $(\exists y) Lxy$  is not a sentence,  $x$  occurs free
- (c)  $(Gxa \& Gya)$  is not a sentence,  $x$  and  $y$  occur free ( $a$  is a constant).

2'. Someone lost his or her book. Anyone who lost the book will be unable to read the chapter. Therefore, someone will be unable to read the chapter.

Let  $Lx$ -- $x$  has lost the book,  $Ux$ -- $x$  is unable to read the chapter. Then we have  $(\exists x) Lx, (\forall x) (Lx \rightarrow Ux) \vdash (\exists x) Ux$ .

1	1. $(\exists x) Lx$		A
2	2. $(x) (Lx \rightarrow Ux)$		A
3	3. $Lc$		A
2	4. $(Lc \rightarrow Uc)$	2	UE
2,3	5. $Uc$	3,4	MPP
2,3	6. $(\exists x) Ux$	5	EI
1,2	7. $(\exists x) Ux$	1,3,6	EE

3. Letting P stand for the relation (among people) of parent ( $Pxy$  says that y is a parent of x), translate the following:

- (a) Everyone has a parent.  $(x) (\exists y) Pxy$   
 (b) Not everyone is a parent.  $\neg(y) (\exists x) Pxy$   
 (c) No-one is the parent of everyone.  $(y) \neg(x) Pxy$   
 (d) Everyone has exactly two parents.  $(x) (\exists y) (\exists z) (\neg y = z \ \& \ (u) (Pxu \leftrightarrow (u = y \vee u = z)))$

4. Prove the following, using only the 16 primitive rules of inference.

(a)  $(x) Fx, (x) (Fx \rightarrow Gx) \vdash (x) Gx$

1	1. $(x) Fx$		A
2	2. $(x) (Fx \rightarrow Gx)$		A
1	3. $Fc$	1	UE
2	4. $(Fc \rightarrow Gc)$	2	UE
1,2	5. $Gc$	3,4	MPP
1,2	6. $(x) Gx$	5	UI

(b)  $(x) (y) Gxy \vdash (x) Gxx$

1	1. $(x) (y) Gxy$		A
1	2. $(y) Gcy$	1	UE
1	3. $Gcc$	2	UE
1	4. $(x) Gxx$	3	

(c)  $(\exists x) (y) Rxy \vdash (y) (\exists x) Rxy$

1	1. $(\exists x) (y) Rxy$		A
2	2. $(y) Ray$		A (use EE)
2	3. $Rab$	2	UE
2	4. $(\exists x) Rxb$	3	EI
2	5. $(y) (\exists x) Rxy$	4	UI
1	6. $(y) (\exists x) Rxy$	1,2,5	EE

(d)  $((x) Fx \vee (x) Gx) \vdash (x) (Fx \vee Gx)$

1	1. $((x) Fx \vee (x) Gx)$		A
2	2. $(x) Fx$		A (use $\vee E$ )
2	3. $Fc$	2	UE
2	4. $(Fc \vee Gc)$	3	$\vee I$
5	5. $(x) Gx$		A
5	6. $Gc$	5	UE
5	7. $(Gc \vee Fc)$	6	$\vee I$
1	8. $(Gc \vee Fc)$	1,2,4,5,7	$\vee E$
1	9. $(x) (Gx \vee Fx)$	8	UI

(e)  $(x) Fx \vdash (\exists x) Fx$ 

1	1.	$(x) Fx$	A	
1	2.	$Fc$	1	UE
1	3.	$(\exists x) Fx$	2	EI

(f)  $(\exists x) Fx, (x) (Fx \rightarrow Gx) \vdash (\exists x) Gx$ 

1	1.	$(\exists x) Fx$	A	
2	2.	$(x) (Fx \rightarrow Gx)$	A	
3	3.	$Fc$	A	(use EE)
2	4.	$(Fc \rightarrow Gc)$	2	UE
2,3	5.	$Gc$	3,4	MPP
2,3	6.	$(\exists x) Gx$	5	EI
1,2	7.	$(\exists x) Gx$	1,3,6	EE

(g)  $\vdash (x) x = x$ 

	1.	$c = c$	=I
	2.	$(x) x = x$	1 UI

(h)  $Fa, a = b, b = c \vdash Fc$ 

1	1.	$Fa$	A
2	2.	$a = b$	A
3	3.	$b = c$	A
1,2	4.	$Fb$	2,1 =I
1,2,3	5.	$Fc$	4,3 =I

(i)  $\vdash (x) (y) ((Fx \& x = y) \rightarrow Fy)$ 

1	1.	$(Fc \& c = d)$	A
1	2.	$Fc$	1 &E
1	3.	$c = d$	1 &E
1	4.	$Fd$	2,3 =E
	5.	$((Fc \& c = d) \rightarrow Fd)$	1,4 CP
	6.	$(y) ((Fc \& c = y) \rightarrow Fy)$	5 UI
	7.	$(x) (y) ((Fx \& x = y) \rightarrow Fy)$	6 UI

5. (a) 1 1.  $Em$  A  
 1 2.  $(x) Ex$  1 UI

line 2 not justified because m is not "arbitrary"

(b) 1 1.  $(\exists x) \neg x = c$  A  
 1 2.  $(x) (\exists x) \neg x = x$  1 UI

line 2 not justified because  $(x) \neg x = c$  is not the result of replacing the free occurrences of x in  $(\exists x) \neg x = x$  by c; also, c is not arbitrary.

(c) 1 1.  $(x) (Fx \rightarrow \neg c = x)$  A  
 1 2.  $(\exists x) (x) (Fx \rightarrow \neg x = x)$  1 EI

line 2 not justified because  $(x) (Fx \rightarrow \neg c = x)$  is not the result of replacing the free occurrences of x in  $(x) (Fx \rightarrow \neg x = x)$  by c