Math 112 Beginning Logic, Spring 1997

Instructor: Julia Knight
Office: 233 CCMB
Hours: MWF 3-4 (tentatively)

MWF 1:55-2:45 First day, 119 Haggar, after that, 282 Galvin

Text: E. J. Lemmon, Beginning Logic (with supplementary material later)

The book sounds as if it was written for British schoolboys around the turn of the century, to sit beside leather-bound books on rhetoric and Latin grammar. The tone may be off-putting, but the book has the right level. It does not assume any mathematical background.

Coursework and Grading:
In-class exams, three such, 15% each
Final, 30% (according to university rules)
Homework, 15% (done in groups, more about that in a moment)
Paper, talk 10% (on material beyond text, more about that later)
Dates
Exam I: Feb. 7
Exam II: March 5 (Wed. before Spring break)
Papers: due March 26 (Wed. before Easter)
Exam III: April 11
Talks: April 21-25
Final: May 5, 8:00-10:00 (first day of finals)

I'll assign homework regularly. Some of the homework will be challenging, constructing proofs. I want you to work in groups. I think that you will find that working together, talking about the material helps you understand it better. Moreover, you will almost certainly be working in groups when you take a regular job.

Assign groups of 3-4.

The first assignment will be to read through p. 8 in the text, and to talk with the members of your group about why you signed up for the course and what you hope to get out of it. I would like each group to turn in a short (< 1 page) statement, jointly written, summarizing this discussion. (Your group may designate a scribe, or each write a paragraph, but all should read and approve the whole before you turn it in on Friday. If you use a computer, it will be easy to fold separate paragraphs together, or make changes.)

In the remaining time, I'd like to say a little about what logic is.

What do we do in logic ?

I. Analyze arguments

We say what is a "sound" argument.

Argument starts with some premises (assumptions) and arrives at a conclusion. These statements may involve mathematical objects such as numbers, or they may involve family relationships or historical facts.

In logic, we look at the form of the argument. We try to say when the form guarantees that anyone who accepts the premises has to accept the conclusion.

Example 1 (famous)

Premises:

All men are mortal.

Socrates is a man.

Conclusion:

Socrates is mortal.

Example 2

Premises:

For all natural numbers n, $1+2+3+...+n = n(n+1)/2$

7 is a natural number

Conclusion:

$1+2+3+4+5+6+7 = 7.8/2 = 28$

Examples 1 and 2 have the same form (syllogism). I think you believe both to be sound.

The next example is more complicated. I won't tell you whether it is sound.

Example 3

Premises:

If the river floods, then our entire wheat crop will be destroyed. The river will flood if there is an early thaw. In any case, there will be heavy rains later in the summer.

Conclusion:

If there is an early thaw, our entire community will be bankrupt unless there are heavy rains later in the summer. Part of the difficulty in analyzing this argument is the length of the sentences. It would be easier to analyze something short, of the same form. Another difficulty is that some words may be hard to interpret. This leads to something else we do in logic.

II. Develop formal languages

Formal languages let us examine the form of an argument, or of a single statement. Translating an argument from English into an apropriate formal language makes the form transparent. We remove ambiguities. I and II are the parts of logic that we shall concentrate on for most of the course. We shall discuss some formal languages, and formal system of proof. Homework will involve translating from English into an appropriate formal language, and analyzing and constructing proofs in formal system.

Students who took similar course in the past, arts and letters students, felt that this kind of exercise should improve their ability to analyze complex material, and their ability write to make a point. There are other kinds of problems, other areas of logic.

III. Describe what is (or is not) computable

What can be done by a machine ? If there is an algorithm (method which could be turned into a computer program) for doing something, you could show this by describing the algorithm. For example, there is an algorithm for deciding whether a natural number n is a prime. Divide n by 2, 3, 4, ..., n-1. If you don't come to a number which divides n, then n is prime. There is an algorithm for deciding whether a polynomial equation in many variables has a solution in real numbers. No such algorithm for solution in integers.

Example of polynomial equation: $7x^2y^3z^5 - 13xyz^2 + 5x^{16}z - 2xz^4 = 0$
American logician, Julia Robinson, reduced problem to something else. Finished by Russian logician, Yuri Matijasevich.

IV. Develop theory of sets

Set theory serves as foundation for mathematics. Discuss the sizes of sets, both finite and infinite. What makes one infinite set larger than another ?

V. Analyze expressibility, definability

Consider the relation between formal languages and real situations, or mathematical structures. What can you say, in a particular setting, using a particular language ? Using a natural language for talking about numbers, in real numbers cannot define or say anything very complicated. In rationals, situation is quite different.

How old is logic ? How did it develop ? Certainly, there were proofs, rules for such, tied to development of geometry in Egypt. Fact that Nile flooded and removed markers between plots of land led to squabbles that could only be settled by sound reasoning from universally accepted geometric premises and observations on distant landmarks.

Formal languages are more recent. Some of the motivation for study of formal languages came from paradoxes. These also motivated more systematic study of sets.

Examples of Paradoxes

1. Liar paradox (ancient Greece)
A man says "I am lying".
If the man is lying, then he is telling the truth, while if he is telling the truth, then he is lying.

2. Russell's paradox (around 1900)
Intuitively, we think of a set as any collection of objects (set of people in South Bend, set of integers).

Let A be the set of all sets X such that X does not belong to itself. If A belongs to A, then A does not belong to A, and if A does not belong to A, then A belongs to A.

3. Richard's Paradox
Some English phrases denote real numbers. For example, "the ratio between the circumference and the diameter of a circle" denotes the number pi. We can order the phrases, shortest first, those of same length lexicographically (as in dictionary). Let r_1,r_2,r_3,... be the numbers denoted by 1st, 2nd, 3rd phrase, in order. Now let r be the real number with 1 in the nth decimal place if r_n does not have 1 there and 2 otherwise. r may be .12211122122211.... Then r does not equal r_n for all n, but r is described by an English phrase.

Logicians found ways around the paradoxes by developing formal languages, and formalizing set theory. The ideas behind the paradoxes were used in sound arguments.

1. The idea of Russell's paradox was used to show that the set of subsets of a given set is always larger than the set itself.

2. The idea of the Liar paradox was used to show that there is no algorithm for deciding which sentences are true of the natural numbers with + and ..

3. The idea of Richard's paradox was used to show that the set of

points on a line is larger than the set of integers. The paper that you write will be on material from logic, not from the text, something you decide to learn about and explain, possibly something on non-computable functions, or set size. I'll say more about this later on.