Brief Article

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Math 119: Calculus Name:__________________________________ **Exam III** Tutorial

Instructor:__________________________________ *December 6, 1994* Tutorial

Section:__________________________________

Calculators are not allowed. Hand in this answer page only. Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 19 multiple choice questions, worth 5 points each. An additional 5 points will be given for your correct tutorial section number.

You are taking this exam under the honor code.

Let $f(x) = \dfrac{\cos^2 x}{\sin x}$. Which of the following statements is true about the limits as $x \to \pi$ from the left and from the right?

$\lim_{x\to\pi-} f(x) = +\infty$, $\lim_{x\to\pi+} f(x) = -\infty$. $\lim_{x\to\pi-} f(x) = +\infty$, $\lim_{x\to\pi+} f(x) = +\infty$. $\lim_{x\to\pi-} f(x) = -\infty$, $\lim_{x\to\pi+} f(x) = -\infty$. $\lim_{x\to\pi-} f(x) = -\infty$, $\lim_{x\to\pi+} f(x) = +\infty$. $\lim_{x\to\pi-} f(x) = 1$, $\lim_{x\to\pi+} f(x) = -1$.

Let

$$f(x) = \begin{cases} x^2+1, & \text{if } 0 \le x \le 1; \\ 2x, & \text{if } 1 \le x \le 2. \end{cases}$$

Find $A_0^x(f)$ for x satisfying $1 \le x \le 2$. $A_0^x(f) = x^2 + \frac{1}{3}$ $A_0^x(f) = x^2$ $A_0^x(f) = x^2 + \frac{4}{3}$ $A_0^x(f) = \frac{1}{3}x^3 + x$ $A_0^x(f) = \frac{1}{3}x^3 + x + \frac{4}{3}$

Consider the function

$$f(x) = 6x^2 + 1$$

Find the area under the graph of $f(x)$ between $x = -1$ and $x = 2$. 21 18 15 9 36

Consider the function

$$f(x) = x + 1$$

Find $\mathbf{U}_0^4(P, f)$ (the upper sum) if P partitions the interval $[0, 4]$ into four equal segments. 14 10 6 18 12

Let $f(x)$ be a function such that $f(x)$, $f'(x)$ and $f''(x)$ exist for all x, and such that $f''(x) = \dfrac{x^2 - 4}{\sqrt{x^4 + 1}}$. You *don't* have to figure out what $f(x)$ is. But which of the following is true about the graph of $f(x)$ at $x = 2$? It *must* have an inflection point at $x = 2$. It *must* have a local maximum at $x = 2$. It *must* have a local minimum at $x = 2$. At $x = 2$ it may have a local minimum or maximum, but it cannot have an inflection point. From the given information it is impossible to tell if it has a local maximum, local
minimum or inflection point at $x = 2$.

Let $f(x) = \dfrac{1}{x} + \dfrac{1}{4}x + 2$. Note that $f'(x) = \dfrac{-1}{x^2} + \dfrac{1}{4}$ and $f''(x) = \dfrac{2}{x^3}$. On what interval(s) is $f(x)$ decreasing? $-2 \le x < 0$ and $0 < x \le 2$ $-\infty < x \le -2$ and $2 \le x < \infty$ $-\infty < x \le -2$ and $0 < x \le 2$ $-2 \le x < 0$ and $2 \le x < \infty$ $-\infty < x < 0$

Suppose $f(x) = 1 + x^2$ for $-3 \le x \le 2$. Take the partition P that divides the interval $[-3, 2]$ into five equal segments, as shown in the diagram below. (Note that the diagram is not drawn perfectly to scale.) Find the lower sum $\mathbf{L}_{-3}^2(P, f)$.

11 24 6 19 9

Let $g(x) = \dfrac{\sqrt{x^2 + 9}}{2x + 7}$. Find the horizontal asymptote(s) of $g(x)$, if there are any. $y = -\dfrac{1}{2}$ and $y = \dfrac{1}{2}$ Only $y = 0$ Only $y = \dfrac{1}{2}$ $g(x)$ has no horizontal asymptotes Only $y = \dfrac{3}{7}$

A ball is thrown upward with a velocity of 16 ft/sec from a height of 32 ft. After how many seconds does it hit the ground? (Remember that the acceleration due to gravity is -32 ft/sec^2.) 2 seconds $\dfrac{1}{2}$ second 1 second 4 seconds 3 seconds

Let $f(x) = \sin(x^2)$. Which of the following statements is true about $f(x)$? (Only one of the statements is true.) $f(x)$ is symmetric about the y-axis. $f(x)$ is symmetric about the origin. $f(x)$ is periodic with period $4\pi^2$. $f(x) \geq 0$ for all x. $\lim_{x\to\infty} f(x) = 0$.

Let $g(x) = \dfrac{x-1}{x+1}$. Find all the asymptotes of $g(x)$. vertical asymptote $x = -1$; horizontal asymptote $y = 1$ vertical asymptote $x = -1$; horizontal asymptote $y = -1$ vertical asymptote $x = 1$; horizontal asymptote $y = 1$ vertical asymptote $x = 1$; horizontal asymptote $y = -1$ vertical asymptotes $x = -1$ and $x = 1$; horizontal asymptote $y = 1$

Let $g(x) = x^4 - 8x^2$. Note that

$$g'(x) = 4x(x^2 - 4)$$
$$g''(x) = 4(3x^2 - 4)$$

At which x, if any, does $g(x)$ have a local maximum? Only at $x = 0$. $g(x)$ has no local maximum. At $x = -2$ and $x = 2$. At $x = -2$, $x = 0$ and $x = 2$. At $x = -\dfrac{2}{\sqrt{3}}$ and $x = \dfrac{2}{\sqrt{3}}$

Let $h(x) = x - 3x^{1/3}$. Note that

$$h'(x) = 1 - x^{-2/3}$$
$$h''(x) = \frac{2}{3}x^{-5/3}$$

On which interval(s) is $h(x)$ concave up? $0 < x < +\infty$ $-\infty < x < 0$ $-\infty < x < 0$ and $0 < x < +\infty$ $-1 < x < 0$ and $0 < x < 1$ $-\infty < x < -1$ and $1 < x < +\infty$

Find the point on the line $y = 3x + 2$ that is closest to the point $(5, 7)$. $(2, 8)$ $\left(\frac{5}{2}, \frac{19}{2}\right)$ $(0, 2)$ $(1, 5)$ $(3, 11)$

A box with a square base and open top must have a volume of 4 ft^3. Find the length x of one side of the base the box if you are to minimize the amount of material used to construct it. (Note you're finding x, not y.)

$x = 2$ $x = 1$ $x = 4$ $x = \dfrac{1}{2}$ $x = \sqrt[3]{4}$

Find an antiderivative of $f(x) = \sin x + \sqrt{x}$. $-\cos x + \dfrac{2}{3}x^{3/2}$ $\cos x + \dfrac{2}{3}x^{3/2}$ $-\cos x - \dfrac{1}{2}x^{-1/2}$ $\cos x - \dfrac{1}{2}x^{-1/2}$ $-\cos x - \dfrac{2}{3}x^{3/2}$

Let $f(x) = x^4 - 4x^3$. Notice that

$$f'(x) = 4x^2(x-3)$$

$$f''(x) = 12x(x-2)$$

Which of the following describes the graph of $f(x)$? local minimum at $x = 3$; inflection points at $x = 0$ and $x = 2$ local minimum at $x = 0$ and $x = 3$; inflection point at $x = 2$ local maximum at $x = 0$; local minimum at $x = 2$; no inflection points local maximum at $x = 0$; local minimum at $x = 2$; inflection point at $x = 3$ local maximum at $x = 0$; local minimum at $x = 3$; inflection points at $x = 0$ and $x = 2$

A particle is moving along a straight line with position function $s(t)$. (Length is measured in feet and time is in seconds.) Its acceleration function is given by $a(t) = 4 - 6t$. You are also given that its initial velocity is $v(0) = 5$ and its initial position is $s(0) = 7$. Find the position after 1 second (i.e. find $s(1)$). 13 feet 6 feet 1 foot -2 feet 5 feet

Let $g(x) = 12 - 3x^2$. Find the area of the portion of the graph of $g(x)$ which is above the x-axis. (Hint: you'll have to figure out where the graph crosses the x-axis.)

32 16 64 48 24