Brief Article

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Math 119: Calculus Name:______________________________ **Final Exam** Tutorial

Instructor:______________________________ *December 19, 1994* Tutorial

Section:______________________________

Calculators are not allowed. Hand in this answer page only. Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 30 multiple choice questions, worth 5 points each.

You are taking this exam under the honor code.

Which of the following is the equation of the line through the point $(1,2)$ and parallel to the line $2x-3y=4$?

$2x-3y=-4$ $2x-3y=6$ $3x-2y=-1$ $3x-2y=0$ $2x+3y=8$

Find $\dfrac{d}{dx}(x^3-2)^{2/3}$ $\dfrac{2x^2}{(x^3-2)^{1/3}}$ $\dfrac{2}{3(x^3-2)^{1/3}}$ $\dfrac{3x^2}{(x^3-2)^{1/3}}$ $2x^2(x^3-2)^{1/3}$ $(3x^2)^{-1/3}$

Where is the graph of the function

$$f(x) \;=\; x^3-6x^2+9x-2$$

concave up, and where is it concave down? concave up on $(2,\infty)$, concave down on $(-\infty,2)$ concave up on $(-\infty,1)$ and $(3,\infty)$, concave down on $(1,3)$ concave up on $(1,3)$, concave down on $(-\infty,1)$ and $(3,\infty)$ concave up on $(0,2)$, concave down on $(-\infty,0)$ and $(2,\infty)$ concave up on $(-\infty,2)$, concave down on $(2,\infty)$

Suppose

$$\lim_{x\to a} f(x)=0 \qquad \text{and} \qquad \lim_{x\to a} g(x)=0\,.$$

Which, if any, of the following statements has to be true about $\lim_{x\to a}\dfrac{f(x)}{g(x)}$? You can't tell from the given information what $\lim_{x\to a} \frac{f(x)}{g(x)}$ is. $\lim_{x\to a} \frac{f(x)}{g(x)}$ does not exist. $\lim_{x\to a} \frac{f(x)}{g(x)} = 0$ $\lim_{x\to a} \frac{f(x)}{g(x)} = 1$ Either $\lim_{x\to a} \frac{f(x)}{g(x)} = \infty \quad \text{or} \quad \lim_{x\to a} \frac{f(x)}{g(x)} = -\infty$

Find $\lim_{h\to 0} \frac{(2h-3)^2-9}{h}$ -12 2 -6 0 -3

Find $\frac{d}{dx}\left(x^2\cos x\right)$ $-x^2\sin x + 2x\cos x$ $x^2\sin x + 2x\cos x$ $-2x\sin x$ $x^2\sin x - 2x\cos x$ $2x\sin x$

Find the slope of the tangent line to the graph of the equation

$$x^2 - y^2 + xy = -1$$

at the point $(1,2)$ $\frac{4}{3}$ $\frac{5}{3}$ $\frac{2}{3}$ 1 3

Find $\int_0^1 (x^2-2x)dx$ $\frac{-2}{3}$ $\frac{-1}{3}$ -1 $\frac{1}{3}$ $\frac{4}{3}$

If $\int f(x)dx = x^3 - 3x^2 + 2x + C$, what is $f'(x)$? $6x-6$ $3x^2-6x+2$ $\frac{x^4}{4} - x^3 + x^2$ x^3-3x^2+2x None of the above

Find the area under the graph of the function

$$f(x) = \sqrt{x}$$

from $x=0$ to $x=4$. $\frac{16}{3}$ 1 $\frac{14}{3}$ $\frac{1}{4}$ $\frac{11}{2}$

Find $\int_0^{2\pi} \sin x \, dx$ 0 2 -2 2π 1

Find $\lim_{x\to\infty} \sqrt{x^2-x+1} - x$ $-1/2$ ∞ 0 1 -1

Let $f(x)$ be a function which is continuous on the interval $[a,b]$, but not necessarily positive on the whole interval. Which of the following statements **must** be true:

I. $f(x)$ has a derivative which is defined at every point of (a,b)

II. $f(x)$ has an antiderivative which is defined at every point of (a,b)

III. $\int_a^b f(x)dx$ is equal to the area between the graph of $f(x)$ and the x-axis.

Only II. Only I. Only III. II. and III. All three statements must be true

Find all the asymptotes of the function $\dfrac{x-1}{(x-2)(x-3)}$. vertical asymptotes $x = 2$, $x = 3$; horizontal asymptote $y = 0$ vertical asymptotes $x = 2$, $x = 3$; no horizontal asymptote vertical asymptotes $x = 2$, $x = 3$; horizontal asymptote $y = 1/2$ vertical asymptote $x = 1$; no horizontal asymptote vertical asymptotes $x = 2$, $x = 3$; horizontal asymptote $y = 1/6$

Suppose f and g are functions satisfying the following conditions: $f(0) = 1$ $f(1) = 0$

$g(0) = 3$ $g(1) = -2$ $f'(0) = 2/3$ $f'(1) = -1/2$ $g'(0) = -5$

$g'(1) = 1/3$ If $h(x) = g(f(x))$, what is $h'(1)$? $5/2$ $-1/6$ $2/9$ $-10/3$ 0

A particle is moving in the positive direction along the x-axis so that its position on the axis at t seconds is given by the formula

$$s(t) = \frac{t^2}{t+1}$$

What is the instantaneous velocity of the particle after one second? 3/4 1/2 2 1 0

What is the natural domain of the function $f(x) = \cot 2x$? All x except $0, \pm\frac{\pi}{2}, \pm\pi, \pm\frac{3\pi}{2}, \pm 2\pi \ldots$ All x except $0, \pm\pi, \pm 2\pi, \pm 3\pi \ldots$ All x except $\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots$ All x except $0, \pm 2\pi, \pm 4\pi, \pm 6\pi \ldots$ All x except $0, \pi, 2\pi, 3\pi \ldots$

Find the radius of the circle given by the equation

$$x^2 + 2x + y^2 + 4y = 4$$

1 2 3 4 5

A water trough is 5 ft long and its ends have the shape of isosceles triangles that are 3 ft high and 6 feet across the top (see diagram). Water is being poured in at a rate of 20 ft^3/min. At a certain instant, the water is rising at a rate of 3 ft/min. How high is the water in the trough at that instant?

2/3 ft 2 ft 1 ft 1/2 ft 3 ft

Which of the following limits represents the slope of the graph of

$$y = \sin x$$

at $x = \frac{\pi}{6}$? $\lim_{h\to 0} \frac{1}{h}\left[\sin\left(\frac{\pi}{6}+h\right)-\frac{1}{2}\right]$ $\lim_{h\to 0} \frac{1}{h}\left[\sin\left(\frac{\pi}{6}+h\right)-\frac{\sqrt{3}}{2}\right]$ $\lim_{h\to 0} \frac{1}{h}\left[\sin(h)-\frac{1}{2}\right]$ $\lim_{h\to 0} \frac{1}{h}\left[\sin\left(\frac{\pi}{6}+h\right)-\sin h\right]$ $\lim_{h\to 0} \frac{\sin h}{h}$

Where is the following function continuous and where is it differentiable (i.e. where does its derivative exist)?

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ x^2 & \text{if } 0 \le x \le 1 \\ x & \text{if } x > 1 \end{cases}$$

continuous everywhere; differentiable everywhere except at $x = 1$ continuous everywhere; differentiable everywhere continuous everywhere except at $x = 0$; differentiable everywhere except at $x = 0$ and $x = 1$ continuous everywhere except at $x = 0$ and $x = 1$; differentiable everywhere except at $x = 0$ and $x = 1$ continuous everywhere; differentiable everywhere except at $x = 0$ and $x = 1$

Which of the following is the upper sum for the function $g(x) = 1 - x^2$ over the interval $[0, 2]$ (i.e. $0 \le x \le 2$) if the interval is divided into 4 equal segments? $\frac{1}{2}\left[1+\frac{3}{4}+0-\frac{5}{4}\right]$ $\frac{1}{2}\left[\frac{3}{4}+0-\frac{5}{4}-3\right]$ $\frac{1}{2}\left[1+\frac{3}{4}+0+\frac{5}{4}\right]$ $\frac{1}{2}\left[1+\frac{3}{4}-\frac{5}{4}-3\right]$ $\frac{1}{2}\left[\frac{1}{2}+1+\frac{3}{2}+2\right]$

Let $f(x) = \sin x$ for $0 \le x \le \pi$. Find $A_0^x(f)$ (the area function) for $0 \le x \le \pi$. $-\cos x + 1$ $-\cos x$ $\cos x$ 2 π

Let

$$f(x) = \begin{cases} \sqrt{x}+1 & \text{if } 0 \le x \le 1 \\ 2 & \text{if } 1 \le x \le 2 \end{cases}$$

Find the area function $A_0^x(f)$ for $0 \le x \le 2$. $A_0^x(f) = \begin{cases} \frac{2}{3}x^{3/2}+x & \text{if } 0 \le x \le 1 \\ 2x-\frac{1}{3} & \text{if } 1 \le x \le 2 \end{cases}$ $A_0^x(f) = \begin{cases} \frac{2}{3}x^{3/2}+x & \text{if } 0 \le x \le 1 \\ 2x+\frac{5}{3} & \text{if } 1 \le x \le 2 \end{cases}$ $A_0^x(f) = \begin{cases} \frac{2}{3}x^{3/2}+x+1 & \text{if } 0 \le x \le 1 \\ 2x+\frac{5}{3} & \text{if } 1 \le x \le 2 \end{cases}$ $A_0^x(f) = \begin{cases} \frac{2}{3}x^{3/2}+x & \text{if } 0 \le x \le 1 \\ 2x & \text{if } 1 \le x \le 2 \end{cases}$ $A_0^x(f) = \begin{cases} \frac{1}{2}x^{-1/2} & \text{if } 0 \le x \le 1 \\ 0 & \text{if } 1 \le x \le 2 \end{cases}$

A circle is expanding in such a way that its radius is growing at a constant rate of 3 ft/sec. At what rate is its area increasing when the radius is 5 ft? 30π ft^2/sec 6π ft^2/sec 10π ft^2/sec 15π ft^2/sec 25π ft^2/sec

Let $f(x) = 6x^{1/3} - \frac{3}{4}x^{4/3}$. Notice that

$$f'(x) = 2x^{-2/3} - x^{1/3} = \left(\frac{1}{x^{2/3}}\right)(2-x)$$

Find all the local maxima and local minima of $f(x)$.

local maximum at $x = 2$; no local minimum local maximum at $x = 2$; local minimum at $x = 0$ local maximum at $x = 2$ and at $x = 0$; no local minimum no local maximum; local minimum at $x = 2$ no local maximum; local minimum at $x = 0$ and at $x = 2$

If $F(x)$ is an antiderivative for $f(x) = x^2 + 2x + 1$ satisfying $F(3) = 25$, what is $F(0)$? 4 2 1 $\frac{7}{3}$ 0

Find the point on the graph of the function $y = \sqrt{x}$ which is closest to the point $(3, 0)$. $\left(\frac{5}{2}, \sqrt{\frac{5}{2}}\right)$ $(4, 2)$ $(1, 1)$ $(2, \sqrt{2})$ $(3, \sqrt{3})$

A rectangle is to be inscribed in a circle of diameter 10 (see diagram). Your mission is to determine the dimensions of the rectangle which maximize its area. What function would you differentiate to solve that problem? (You don't have to figure out the final answer.)

$x\sqrt{100 - x^2}$ $\sqrt{100 - x^2}$ $x^2 + y^2$ $x^2\sqrt{100 - x^2}$ $\frac{x^2}{\sqrt{100 - x^2}}$

Find $\lim_{h \to 0} \frac{\sin(\pi + h)}{2h}$. $-1/2$ $1/2$ 1 -1 $\frac{\pi}{2}$