## Brief Article

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| Math 119: Calculus | Name: |                  | Exam II | Tutorial |
|--------------------|-------|------------------|---------|----------|
| Instructor:        |       | October 31, 1995 |         | Tutorial |
| Section:           |       |                  |         |          |

Calculators are not allowed. Hand in this answer page only. Record your answers to the multiple choice problems by placing an  $\times$  through one letter for each problem on this answer sheet. There are 19 multiple choice questions, worth 5 points each. An additional 5 points will be given for your correct tutorial section number.

## You are taking this exam under the honor code.

let  $f(x) = \sin x + \cos x$ . Find all values of x between 0 and  $2\pi$ , inclusive, where the tangent line to the graph of f(x) is horizontal.  $x = \frac{\pi}{4}, \frac{5\pi}{4}$   $x = \frac{3\pi}{4}, \frac{7\pi}{4}$   $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$   $x = 0, \pi, 2\pi$   $x = \frac{\pi}{2}, \frac{3\pi}{2}$ 

Let 
$$f(x) = \tan(x^2 + 1)$$
. Find  $f'(x)$ .  $2x \sec^2(x^2 + 1) \sec^2(x^2 + 1) \sec^2(2x) \tan(2x) \cdot \sec^2(x^2 + 1) \sec^2(x^2 + 1) + \tan(2x)$ 

If 
$$xy + 1 = y^2$$
, use implicit differentiation to find  $\frac{dy}{dx}$ .  $\frac{-y}{x-2y} \frac{x+y}{2y} = 0$   $\frac{y}{x} \frac{-y-1}{x-2y}$ 

Let  $f(x) = \sin x$ . Find the one hundredth derivative  $f^{(100)}(x)$ .  $\sin x \cos x - \sin x - \cos x$  (100!)  $\sin x$ 

Let 
$$f(x) = x^{10} + x^5 + 3$$
. Find  $f^{(10)}(x)$ . 10! 10! + 5! 10! + 5! + 3! 0 10! + 5! + 3

If a ball is thrown vertically upward with a velocity of 16 ft/sec, then its height after t seconds is  $s = 16t - 16t^2$ . What is the maximum height reached by the ball? 4 ft. 0 ft. 16 ft. 8 ft. 32 ft.

The equation of motion for a certain particle is  $s = t^4 - t^2 + 1$ . What is the equation for the acceleration of the particle?  $12t^2 - 2$   $12t^2$   $4t^3 - 2t$  24t  $4t^3 - 2t + 1$ 

Dave is standing at the edge of a canal directly across from point A. The canal is 8 feet across.

Mary is walking along the canal, away from point A, at a rate of 5 ft/sec. When she is 6 ft away from point A, at what rate is the distance between Dave and Mary increasing?

3 ft/sec 4 ft/sec 2 ft/sec 1 ft/sec 5 ft/sec

A spherical snowball is melting in such a way that its volume is decreasing at a rate of  $4 \text{ cm}^3/\text{min}$ . Recall that the formula for the volume of a sphere is

$$V = \frac{4}{3}\pi r^3$$

where V is the volume and r is the radius of the sphere. Find  $\frac{dr}{dt}$  at the moment when r=5 cm. (Hint: don't forget that the snowball is getting smaller.)  $\frac{-1}{25\pi}$   $\frac{1}{25\pi}$   $\frac{3}{125\pi}$   $\frac{-3}{125\pi}$   $\frac{-1}{125\pi}$ 

Let  $f(x) = x^3 - 12x$ . Find the absolute maximum value of f(x) on the interval [0,3]. (You are looking for a y-value, not an x-value.) 0-9-16 16 2

Let

$$f(x) = \frac{x}{x^2 + 1}$$

Find (all) the critical numbers of f(x), if there are any. x = -1, 1 x = 1 x = -1 there are no critical numbers x = 0

Let  $f(x) = 5x^{2/3} + x^{5/3}$ . It happens to be true that

$$f'(x) = \frac{10 + 5x}{3x^{1/3}}$$

Find the value(s) of x, if any, where f(x) has a local minimum x=0 x=-2,0. x=-2 x=2 f(x) has no local minimum

Let  $f(x) = x^4 - 6x^2$ . Find the interval(s) where f(x) is concave up.  $(-\infty, -1) \cup (1, \infty)$  (-1, 1)  $(-\infty, 1)$  f(x) is concave up everywhere f(x) is not concave up anywhere

Consider the curve  $x=y^2$ . Find the slope of the tangent line at the point (4,2).  $\frac{1}{4}$  4  $\frac{1}{8}$  8  $\frac{1}{2}$ 

Let

$$f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 + 5.$$

Find all the local extrema of f(x). local minimum at x = 1 local maximum at x = 0, local minimum at x = 1 local minimum at x = 0, local maximum at x = 1 there are no local extrema local minimum at x = 0, local minimum at x = 1

Let  $f(x) = \frac{1}{2}x^2 + \sin x$ . Find the inflection points, if any, of f(x) in the range  $0 \le x \le 2\pi$ . there are no inflection points  $x = \frac{\pi}{2}, \frac{3\pi}{2}$   $x = \frac{\pi}{2}, \pi, \frac{3\pi}{2}$   $x = \frac{3\pi}{2}$ 

Let 
$$f(x) = \sqrt[3]{x^4 - x}$$
. Find  $f'(x)$ .  $\frac{4x^3 - 1}{3(x^4 - x)^{2/3}} \frac{1}{3(x^4 - x)^{2/3}} \frac{3(4x^3 - 1)}{(x^4 - x)^{2/3}} \frac{(x^4 - x)^{2/3}}{4x^3 - 1} \frac{4(x^3 - 1)}{3(x^4 - x)^{2/3}}$ 

Consider the curve  $4x^2 + y^2 = 4$ . It is a fact that

$$\frac{dy}{dx} = -4 \cdot \frac{x}{y}$$

Using this fact, find all the points on the curve where the tangent line is horizontal. (0,2) and (0,-2) (1,0) and (-1,0) (0,2) (1,0) (0,0)

An inverted conical tank has radius 4 meters and height 10 meters. It is being filled with water at a rate of 8 m<sup>3</sup>/min. How quickly is the water rising when the tank is half full (i.e. when the height of the water reaches 5 meters)? Recall that the formula for the volume of a cone of radius r and height h is  $V = \frac{1}{3}\pi r^2 h$ .

$$\frac{2}{\pi} \frac{4}{\pi} \frac{\pi}{3} \frac{\pi}{10} 6\pi$$