Brief Article

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Math 119: Calculus Name:________________________________ **Exam III** Tutorial

Instructor:________________________________ *November 30, 1995* Tutorial

Section:________________________________

Calculators are not allowed. Hand in this answer page only. Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 19 multiple choice questions, worth 5 points each. An additional 5 points will be given for your correct tutorial section number.

You are taking this exam under the honor code.

Which of the following is equal to $\sum_{i=1}^{n}(4-3i)$? $4n-\frac{3n(n+1)}{2}$ $4n-\frac{3n(n+1)(2n+1)}{6}$ $4n-\frac{n(n+1)}{2}$ $4n-\frac{3(n+1)(n+2)}{2}$ $4n-\frac{3n(n+1)(2n+1)}{2}$

Which of the following is equal to $\sum_{i=4}^{100} i$? (Note that it says $i=4$, not $i=1$.) 5044 10094 96 10100 97

Find $\lim_{x\to\infty}(\sqrt{x^2+1}-x)$ 0 1 ∞ $-\infty$ -1

Find all the asymptotes of the function $g(x)=\frac{3x+1}{2x-1}$. horizontal asymptote at $y=\frac{3}{2}$; vertical asymptote at $x=\frac{1}{2}$ horizontal asymptotes at $y=\frac{3}{2}$ and $y=-\frac{3}{2}$; vertical asymptote at $x=\frac{1}{2}$ horizontal asymptotes at $y=\frac{3}{2}$ and $y=-\frac{3}{2}$; vertical asymptote at $x=-\frac{1}{3}$ horizontal asymptote at $y=\frac{3}{2}$; vertical asymptote at $x=2$ horizontal asymptote at $y=\frac{3}{2}$; vertical asymptote at $x=-\frac{1}{3}$

Let

$$f(x) = \begin{cases} 2-x & \text{if } 0 \le x \le 1; \\ x & \text{if } 1 \le x \le 2. \end{cases}$$

Find $A_0^x(f)$ (the area function). $A_0^x(f) = \begin{cases} 2x - \frac{1}{2}x^2 & \text{if } 0 \le x \le 1; \\ \frac{1}{2}x^2 + 1 & \text{if } 1 \le x \le 2 \end{cases}$ $A_0^x(f) = \begin{cases} 2x - \frac{1}{2}x^2 & \text{if } 0 \le x \le 1; \\ \frac{1}{2}x^2 & \text{if } 1 \le x \le 2 \end{cases}$ $A_0^x(f) = \begin{cases} 2x - \frac{1}{2}x^2 & \text{if } 0 \le x \le 1; \\ \frac{1}{2}x^2 + \frac{1}{2} & \text{if } 1 \le x \le 2 \end{cases}$ $A_0^x(f) = \begin{cases} 2x - \frac{1}{2}x^2 & \text{if } 0 \le x \le 1; \\ \frac{1}{2}x^2 - 1 & \text{if } 1 \le x \le 2 \end{cases}$ $A_0^x(f) = \begin{cases} 2x - \frac{1}{2}x^2 & \text{if } 0 \le x \le 1; \\ \frac{1}{2}x^2 - \frac{1}{2} & \text{if } 1 \le x \le 2 \end{cases}$

Let $f(x) = x^3 + x + 1$. Which of the following statements are true about this function?

I. f has no critical numbers. II. f has no inflection points. III. f has no asymptotes (either horizontal or vertical).

(Recall that a critical number is a value c such that either $f'(c) = 0$ or $f'(c)$ does not exist. An inflection point is a place where f changes from being concave up to being concave down, or vice versa.)

I. and III. only I. only II. only III. II. and III.

Let $f(x) = 8x^3 - x^4$. Notice that

$$f'(x) = 4x^2(6-x)$$

$$f''(x) = 12x(4-x)$$

Which of the following describes the graph of $f(x)$? local maximum at $x = 6$; inflection points only at $x = 0$ and $x = 4$ local minimum at $x = 0$ and $x = 6$; inflection point only at $x = 4$ local minimum at $x = 0$; local maximum at $x = 6$; no inflection points local minimum at $x = 0$; local maximum at $x = 6$; inflection points only at $x = 0$ and $x = 4$ local maximum at $x = 6$; inflection point only at $x = 4$ and $x = 2$

A farmer wants to build a rectangular enclosure for his pigs along the side of his barn, which is 40 ft. long. He has 100 ft. of fencing, and he wants to enclose as large an area as possible. Of course he only has to use the fencing for three sides of the enclosure since the barn serves as the fourth side, but that means that the barn side of the enclosure (and hence also the side opposite it) is forced to be *at most* 40 ft. (it could be less). What is the largest area that he can enclose under these conditions?

1200 ft. 1250 ft. 2400 ft. 800 ft. 1111.11 ft.

Find an antiderivative of $f(x) = \cos x - \sqrt[3]{x}$. $\sin x - \frac{3}{4}x^{4/3}$ $-\sin x - \frac{3}{4}x^{4/3}$ $-\sin x + \frac{1}{3}x^{-2/3}$ $\sin x + \frac{3}{4}x^{4/3}$ $\sin x - \frac{1}{3}x^{-2/3}$

A particle moves along a straight line with position function $s(t)$. (Length is measured in feet and time is in seconds.) Its acceleration function is given by $a(t) = 2 - 12t$. You are also given that its initial velocity is $v(0) = 3$ and its initial position is $s(0) = 2$. Find the position after 1 second (i.e. find $s(1)$). 4 ft. 3 ft. 2 ft. 1 ft 5 ft.

Let $f(x) = x + 2$ on the interval $[1, 4]$. If the interval is divided into 3 equal subintervals (i.e. $n = 3$) and $x_i^* =$ left endpoint, find the sum of the areas of the corresponding approximating rectangles. 12 15 8 13 $\frac{27}{2}$

The following limit represents the area of the region below the graph of a certain function $f(x)$ and above a certain interval $[a, b]$:

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{4}{n} \sqrt{\cos(1 + \frac{4i}{n})}$$

Find $f(x)$ and $[a, b]$. $f(x) = \sqrt{\cos x}$, $[a, b] = [1, 5]$ $f(x) = 4\sqrt{\cos x}$, $[a, b] = [1, 2]$ $f(x) = 4\sqrt{\cos(1 + x)}$, $[a, b] = [1, 5]$ $f(x) = \sqrt{\cos x}$, $[a, b] = [0, 4]$ $f(x) = \sqrt{\cos(1 + x)}$, $[a, b] = [1, 5]$

Let x and y be two numbers (not necessarily positive) whose difference is 100. If the product of x and y is the minimum possible, what is the sum of x and y? 0 100 -100 102 200

Find the area of the region below the graph of $y = \sin x$ and above the x-axis, from $x = 0$ to $x = \pi$.

2 1 π $\frac{\pi}{2}$ $\frac{\pi}{3}$

Let $f(x) = \frac{x+1}{|x-1|}$. Which of the following statements is true about the limits as $x \to 1$ from the left and from the right? $\lim_{x\to 1-} f(x) = +\infty$; $\lim_{x\to 1+} f(x) = +\infty$ $\lim_{x\to 1-} f(x) = +\infty$; $\lim_{x\to 1+} f(x) = -\infty$ $\lim_{x\to 1-} f(x) = -\infty$; $\lim_{x\to 1+} f(x) = +\infty$ $\lim_{x\to 1-} f(x) = -\infty$; $\lim_{x\to 1+} f(x) = -\infty$ $\lim_{x\to 1-} f(x) = 1$; $\lim_{x\to 1+} f(x) = 1$

Let $f(x)$ be a function such that $f(x)$ and $f'(x)$ both exist for all x, and such that

$$f'(x) = \frac{4 - x^2}{\sqrt{x^4 + 1}}$$

You *don't* have to figure out what $f(x)$ is. But which of the following is true about the graph of $f(x)$ at $x = 2$? It *must* have a local maximum at $x = 2$. It *must* have an absolute maximum at $x = 2$. It *must* have a local minimum at $x = 2$. It *must* have an absolute minimum at $x = 2$. It *must* have an inflection point at $x = 2$.

Given that the graph of f passes through the point $(1, 4)$ and that the slope of the tangent line at $(x, f(x))$ is $4x - 1$, find $f(2)$. 9 6 4 7 2

A box with an open top is to be constructed from a square piece of cardboard, 6 ft wide, by cutting out a square from each of the four corners and bending up the sides (along the dashed lines in the picture below). Find the largest volume that such a box can have.

16 ft^3 32 ft^3 8 ft^3 4 ft^3 12 ft^3

Find $\lim_{x\to\infty} \frac{1 - x^2}{2 + x^2}$ -1 1 ∞ $-\infty$ $\frac{1}{2}$