Brief Article

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Math 119: Calculus Name:____________________________ **Final Exam** Tutorial

Instructor:____________________________ *December 15, 1995* Tutorial

Section:____________________________

Calculators are not allowed. Hand in this answer page only. Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 29 multiple choice questions, worth 5 points each. An additional 5 points will be given for your correct tutorial section number.

You are taking this exam under the honor code.

Find the equation of the line through the points $(3, 2)$ and $(2, 5)$. $3x+y = 11$ $3x+y = 9$ $x+3y = 9$ $x + 3y = 11$ $-3x + y = -7$

Find the domain of the function $f(x) = \sqrt{x^2 - 1}$. (Recall that $A \cup B$ means all x which are either in A or B or both.)

$(-\infty, -1] \cup [1, \infty)$ $[-1, 1]$ all x $[1, \infty)$ $[0, \infty)$

Is the following the equation of a circle? If so, what is its center?

$$x^2 - 2x + y^2 + 2y - 3 = 0$$

yes, center $(1, -1)$ no, it's not a circle yes, center $(-1, 1)$ yes, center $(1, 1)$ yes, center $(-1, -1)$ Consider the graph of the function $f(x) = x^2$. (See the diagram to the right.) What is the slope of the secant line joining the points on the graph corresponding to $x = 1$ and $x = 1 + h$? $2 + h$ 2 $\frac{(1+h)^2}{h}$ $(1 + h)^2$ $\frac{1}{(1+h)^2}$

Find $\lim\limits_{t\to 1} \frac{t^3 - t}{t - 1}$. 2 this limit does not exist 1 0 $\frac{1}{2}$

Suppose f and g are functions satisfying the following conditions:

$$f(0) = 2 \qquad f'(0) = 3$$
$$g(0) = 4 \qquad g'(0) = 5$$

Let $h(x) = \dfrac{f(x)}{g(x)}$. Find $h'(0)$. $\frac{1}{8}$ can't be determined from the given information $\frac{11}{8}$ $\frac{3}{5}$ $-\frac{1}{8}$

For what value of the constant c is the following function f continuous on $(-\infty, \infty)$:

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x \le 1; \\ cx + 2 & \text{if } x > 1. \end{cases}$$

0 1 2 -1 $\frac{1}{2}$

Let θ be an angle between $\frac{\pi}{2}$ and π, such that $\sin\theta = \frac{1}{4}$. Find $\cos\theta$. (Hint: don't forget I said $\frac{\pi}{2} \le \theta \le \pi$.) $-\frac{\sqrt{15}}{4}$ $\frac{\sqrt{15}}{4}$ $\frac{\sqrt{3}}{4}$ $-\frac{\sqrt{3}}{4}$ $-\frac{1}{\sqrt{15}}$

Convert 210° (degrees) to radians. $\frac{7\pi}{6}$ $\frac{5\pi}{6}$ $\frac{5\pi}{2}$ $\frac{3\pi}{2}$ $\frac{11\pi}{6}$

Find $\lim_{t \to 0} \dfrac{\sin^2 4t}{t^2}$. 16 4 2 1 8

Find $\frac{d}{dx}\csc(x^2)$. $-2x\csc(x^2)\cot(x^2)$ $-\csc(2x)\cot(2x)$ $2x\csc x - x^2\csc x\cot x$ $-\csc(x^2)\cot(x^2)$ $2x\csc(x^2)$

Find $\frac{d}{dx}\sqrt{1+x^2}$. $\frac{x}{\sqrt{1+x^x}}$ $\frac{1}{2\sqrt{1+x^2}}$ $\frac{2x}{\sqrt{1+x^x}}$ $\frac{x^2}{\sqrt{1+x^x}}$ $\frac{x}{2\sqrt{1+x^x}}$

Find $\frac{dy}{dx}$ by implicit differentiation: $x^2 - xy + y^3 = 8$. $\frac{-2x+y}{-x+3y^2}$ $\frac{-2x-y}{-x+3y^2}$ $\frac{8-2x+y}{-x+3y^2}$ $\frac{8-2x-y}{-x+3y^2}$ $\frac{-2x-y}{-x+3y}$

Let $f(x)$ be some function whose first derivative is $f'(x) = \dfrac{x}{x^2+1}$. Find $f''(1)$. (Note that's $f''(1)$, not $f'(1)$.) 0 $\frac{1}{2}$ 1 $\frac{1}{4}$ 4

Let $f(x) = x^{100} + x^{50} + x^5 + 1$. Find $f^{(50)}(0)$. $50!$ $\frac{100!}{50!} + 50!$ $100! + 50!$ $100! + 50! + 5!$ 0

A particle is moving along the x-axis so that its position on the x-axis is given by the formula $s(t) = \dfrac{t-1}{t+1}$, where we are only interested in $t \ge 0$. Notice that

$$s'(t) = \frac{2}{(t+1)^2} \qquad s''(t) = \frac{-4}{(t+1)^3}$$

(you don't have to verify these facts). Which of the following is/are true:

I. The particle is always moving in the positive direction. II. The particle is always decelerating (i.e. slowing down). III. The particle is getting closer and closer to the position $x = 1$, but never quite reaches it.

all three are true only I. and II. are true only I. is true only I. and III. are true only II. and III. are true

A large spherical balloon is being inflated in such a way that its volume is increasing at a rate of 100 ft^3/sec. At what rate is its radius increasing when the radius is 5 ft? You may use the fact that the volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$.

$\frac{1}{\pi}$ ft/sec $\frac{5}{\pi}$ ft/sec π ft/sec $\frac{500\pi}{3}$ ft/sec $\frac{1}{5\pi}$ ft/sec

A particle is moving with the following data:

$$a(t) = 12t^2 \qquad v(0) = 3 \qquad s(0) = 1$$

Find the position of the particle at time $t = 1$. 5 1 24 2 4

Let $f(x) = 1 + (x + 1)^2$. Find the absolute maximum and minimum values of $f(x)$ on the interval $[-2, 2]$. absolute maximum $= 10$ absolute minimum $= 1$ absolute maximum $= 10$ absolute minimum $= 2$ absolute maximum $= 9$ absolute minimum $= 0$ no absolute maximum absolute minimum $= 1$ absolute maximum $= 10$ no absolute minimum

Let $f(x)$ be some function whose derivative is

$$f'(x) = \frac{3x(x-4)}{(x+1)^2}$$

Find the intervals on which $f(x)$ is increasing or decreasing. (You don't have to figure out what $f(x)$ is. Recall that $A \cup B$ means all x which are either in A or B or both.) increasing on $(-\infty, -1) \cup (-1, 0] \cup [4, +\infty)$ decreasing on $[0, 4]$ increasing on $(-1, 0] \cup [4, +\infty)$ decreasing on $(-\infty, -1) \cup [0, 4]$ increasing on $(-\infty, -1) \cup [0, 4]$ decreasing on $(-1, 0] \cup [4, +\infty)$ increasing on $[0, 4]$ decreasing on $(-\infty, -1) \cup (-1, 0] \cup [4, +\infty)$ increasing on $(-\infty, -1) \cup (-1, 0]$ decreasing on $[0, +\infty)$

Let $f(x) = (x^2 - 1)^2$. Notice that

$$f'(x) = 4x(x+1)(x-1) \qquad \text{and} \qquad f''(x) = 4(\sqrt{3}x - 1)(\sqrt{3}x + 1).$$

(You don't have to verify these facts. Note that when I write $\sqrt{3}x$, the x is not inside the square root sign.) On what interval(s) is f concave up? (Recall that $A \cup B$ means all x which are either in A or B or both.) $(-\infty, \frac{-1}{\sqrt{3}}) \cup (\frac{1}{\sqrt{3}}, +\infty)$ $(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$ it's concave up everywhere it's never concave up $(-1, 0) \cup (1, +\infty)$

Find $\lim_{x\to-\infty} \frac{\sqrt{x^2+1}}{2x-5}$ (Note that's a $-\infty$, not a $+\infty$.) $-\frac{1}{2}$ $\frac{1}{2}$ $\frac{5}{2}$ $-\frac{1}{5}$ $\frac{1}{5}$

If 4800 cm^3 of material is available to make a box with a square base and an open top, find the length of one side of the base of the box with largest possible volume. (I.e. find the value of x in the diagram below which maximizes the volume.)

40 cm 20 cm 100 cm 50 cm 25 cm

Write the following sum in sigma notation:

$$2x + 3x^2 + 4x^3 + \cdots + nx^{n-1}$$

$\sum_{i=1}^{n-1}(i+1)x^i$ $\sum_{i=1}^{n}(i+1)x^i$ $\sum_{i=1}^{n}(i)x^i$ $\sum_{i=1}^{n}(i)x^{i-1}$ $\sum_{i=2}^{n}(i)x^i$

Let

$$f(x) = \begin{cases} x+1 & \text{if } 0 \le x \le 1; \\ 2x & \text{if } 1 \le x \le 2. \end{cases}$$

Find $A_0^x(f)$ (the area function). $A_0^x(f) = \begin{cases} \frac{1}{2}x^2 + x & \text{if } 0 \le x \le 1; \\ x^2 + \frac{1}{2} & \text{if } 1 \le x \le 2. \end{cases}$ $A_0^x(f) = \begin{cases} \frac{1}{2}x^2 + x - \frac{1}{2} & \text{if } 0 \le x \le 1; \\ x^2 & \text{if } 1 \le x \le 2. \end{cases}$ $A_0^x(f) = \begin{cases} \frac{1}{2}x^2 + x & \text{if } 0 \le x \le 1; \\ x^2 & \text{if } 1 \le x \le 2. \end{cases}$ $A_0^x(f) = \begin{cases} \frac{1}{2}x^2 + x & \text{if } 0 \le x \le 1; \\ x^2 - \frac{1}{2} & \text{if } 1 \le x \le 2. \end{cases}$ $A_0^x(f) = \begin{cases} \frac{1}{2}x^2 + x + \frac{1}{2} & \text{if } 0 \le x \le 1; \\ x^2 + 1 & \text{if } 1 \le x \le 2. \end{cases}$

Let $f(x) = 9 - x^2$ on the interval$[-2,3]$. If the interval is divided into 5 equal subintervals (i.e. $n = 5$) and $x_i^* =$ right endpoint, find the sum of the areas of the corresponding approximating rectangles. 30 35 44 $\frac{100}{3}$ 25

Evaluate the following integral by interpreting it in terms of areas: $\int_0^2 \sqrt{4-x^2}$ π 2π $\frac{\pi}{2}$ 4π $\frac{\pi}{4}$

A streetlight is at the top of a 15 ft. pole. A man 6 ft. tall walks away from the pole at a rate of 4 ft/sec. At what rate is his shadow lengthening when he is 20 ft. from the base of the pole? $\frac{8}{3}$ ft/sec 6 ft/sec $\frac{4}{3}$ ft/sec 8 ft/sec $\frac{8}{5} ft/sec$

Find the distance between the points $(-2,3)$ and $(6,-3)$. 10 100 4 $\sqrt{52}$ $\sqrt{14}$

I enjoyed teaching this class. Have a great Christmas vacation, and enjoy the Orange Bowl. See you next semester.