Brief Article

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Math 119: Calculus Name:______________________________ **Final Exam**

Instructor:______________________________ *December 16, 1996* Time of MWF

class:______________________________

Calculators are not allowed. Hand in this answer page only. Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 30 multiple choice questions, worth 5 points each.

You are taking this exam under the honor code.

Let

$$f(x) = \begin{cases} \frac{x^2-1}{x-1}, & \text{if } x \neq 1; \\ c, & \text{if } x = 1. \end{cases}$$

For what value of c is $f(x)$ continuous for all x? 2 0 1 -1 -2

Suppose you know that $0 \leq x \leq \dfrac{\pi}{2}$ and $\sin^2 x = \dfrac{1}{2}$. Find $\tan x$.

1 -1 $\dfrac{1}{\sqrt{3}}$ $\sqrt{3}$ $\dfrac{\sqrt{2}}{2}$

Find $\displaystyle\lim_{x\to 0} \frac{\sin^2(5x)}{\sin^2(4x)}$

$\dfrac{25}{16}$ $\dfrac{16}{25}$ $\dfrac{5}{4}$ $\dfrac{4}{5}$ The limit does not exist.

If $f(x) = \tan x$, find the slope of the tangent line to the graph of $y = f(x)$ at $x = \dfrac{5\pi}{4}$.

2 -2 1 -1 0

Let $f(x) = \sin(\cos x)$. Find $f'(\frac{\pi}{2})$.

-1 0 1 $\frac{\sqrt{2}}{2}$ $\frac{1}{2}$

If $xy + y^2 + x^2 = y^3$, find $\frac{dy}{dx}$.

$\frac{-y-2x}{x+2y-3y^2}$ $\frac{x+y}{y^2}$ $\frac{-x-y}{x+y-3y^2}$ $\frac{-x-2y}{x+3y^2}$ $\frac{-2x}{x-3y^2}$

Let $g(x) = x + \frac{1}{x}$. Find $g^{(4)}(x)$ (the fourth derivative).

$\frac{24}{x^5}$ $1 - \frac{24}{x^5}$ $x^4 + \frac{24}{x^4}$ $x^5 - \frac{24}{x^5}$ $\frac{24}{x^4}$

An object dropped from a 100-foot tower on a certain planet has height $h(t) = -4t^2 + 100$ at time t. Find the acceleration due to gravity on that planet.

-8 -4 -2 -16 -32

A particle is moving with constant speed around the circle $x^2 + y^2 = 5$. At the instant when it reaches the point $(1, 2)$, the rate of change of the x-coordinate is -6 units/second. What is the rate of change of the y-coordinate at that instant?

3 units/second -3 units/second $\frac{5}{2}$ units/second $-\frac{5}{2}$ units/second 2 units/second

A cart is being rolled up a 30° ($= \pi/6$ radians) ramp. Its velocity along the ramp is 6 ft/sec. How quickly is its height from the ground increasing when it is 12 feet up the ramp?

3 ft/sec 6 ft/sec 12 ft/sec $6\sqrt{3}$ ft/sec $12\sqrt{3}$ ft/sec

Questions 11 and 12 refer to the function $f(x) = \frac{x+1}{\sqrt{x^2-1}}$. Note that the function has domain $(-\infty, -1) \cup (1, \infty)$.

Exactly two of the following four statements are correct. Which two?

I. $\lim_{x \to 1^+} f(x) = \infty$ **II.** $\lim_{x \to 1^+} f(x) = 0$

III. $\lim_{x \to -1^-} f(x) = -\infty$ **IV.** $\lim_{x \to -1^-} f(x) = 0$

I & **IV** **I** & **III** **II** & **III** **II** & **IV** **I** & **II**

Exactly two of the following four statements are correct. Which two?

I. $\lim_{x \to -\infty} f(x) = 1$ **II.** $\lim_{x \to -\infty} f(x) = -1$

III. $\lim_{x \to \infty} f(x) = 1$ **IV.** $\lim_{x \to \infty} f(x) = -1$

II & **III** **I** & **III** **I** & **IV** **II** & **IV** **III** & **IV**

A stone which is dropped from the top of a certain building takes 3 seconds to hit the ground. How tall is the building? (Recall that the constant acceleration due to gravity is -32 ft/sec.)

144 ft 72 ft 216 ft 432 ft 600 ft

The following is known about the function $f(x)$:

$$f'(x) = 8x^3 + 9x^2 + 8x \; ; \qquad f(1) = 10$$

Find $f(0)$.

1 -1 0 2 -2

Both ends of a 12-inch long piece of wire are bent to form 90° angles, resulting in a "topless" rectangle as shown in the figure. What is the largest area which can be obtained for the rectangle?

18 sq. inches 20 sq. inches 16 sq. inches 22 sq. inches 24 sq. inches

The absolute minimum and maximum values of the function

$$f(x) = x^2 - 6x + 8, \qquad 0 \leq x \leq 8$$

are:

minimum value	-1	minimum value	0	minimum value	-8	minimum value	1	minimum value	1
maximum value	24	maximum value	24	maximum value	8	maximum value	8	maximum value	24

The set of critical points of the function $f(x) = (x^2 - 1)^{4/3}$ consists exactly of the points:

$\{-1, 0, 1\}$ $\{0\}$ $\{-1, 1\}$ $\{0, 1\}$ $\{0, -1\}$

The set of points where the graph of the function

$$f(x) = x^4 - 6x^3 - 24x^2 + 7x + 8$$

is concave upward is:

$(-\infty,-1)\cup(4,\infty)$ $(-\infty,1)\cup(-4,\infty)$ $(-\infty,-1)\cup(-1,4)\cup(4,\infty)$ $(4,\infty)$ none of the above

Which of the following expressions in sigma notation is equivalent to the sum

$$9+16+23+30+37+\cdots+100?$$

$\sum_{n=1}^{14}(2+7n)$ $\sum_{n=0}^{14}(9+7n)$ $\sum_{n=3}^{10}n^2$ $\sum_{n=9}^{100}n$ $\sum_{n=3}^{100}n$

If $f(x)=\frac{x}{x^2+1}$ then what is $f'(x)$?

$\frac{1-x^2}{(x^2+1)^2}$ $\frac{1}{2x}$ $\frac{-1}{x^2}+1$ $\frac{x^2-1}{x^2+1}$ $\frac{3x^2+1}{(x^2+1)^2}$

What is the radius and center of the circle given by the equation $x^2-6x+y^2+4y-3=0$?

radius	4	radius	16	radius	$\sqrt{3}$	radius	3	radius	6
center	$(3,-2)$	center	$(-6,4)$	center	$(3,-2)$	center	$(-3,2)$	center	$(-3,2)$

Evaluate $\lim_{h\to 0}\frac{(1+h)^{1/5}-1}{h}$. Hint: Interpret the limit as the derivative of some function at a particular point.

$\frac{1}{5}$ 0 1 $\frac{-4}{5}$ The limit does not exist

Find the equation of the line passing through the point $(2,3)$ and parallel to the line $y=5x+7$.

$y-3=5(x-2)$ $y+3=5(x+2)$ $y-3=\frac{-1}{5}(x-2)$ $y-2=5(x-3)$ $y-2=7(x-3)$

Find the equation of the tangent line to the graph of the equation $y=\sin(4x)+x+2$ when $x=0$.

$y-2=5x$ $y-5=5x$ $y-2=x$ $y-5=2x$ $y+2=-5x$

Find the intervals on which the function $f(x)=x^3-6x^2+2$ is increasing.

$(-\infty,0)\cup(4,\infty)$ $(-\infty,\infty)$ $(-\infty,4)$ $(0,\infty)$ $(0,4)$

Let

$$f(x)=\begin{cases} x^2+1 & \text{if } x<0\\ 2x+1 & \text{if } 0\le x\le 1\\ x^2+2 & \text{if } x\ge 1.\end{cases}$$

Which of the following statements is true?

$f(x)$ is differentiable at $x = 1$ but not at $x = 0$ $f(x)$ is differentiable at both $x = 0$ and $x = 1$ $f(x)$ is differentiable at $x = 0$ but not at $x = 1$ $f(x)$ is NOT differentiable at both $x = 0$ and $x = 1$ $1 + 1 = 37$

Which one of the following is an antiderivative of $f(x) = x^{-2} + \sin(3x)$?

$-x^{-1} - \frac{\cos(3x)}{3}$ $-x^{-1} + \cos(3x)$ $\frac{-1}{3}x^{-3} + cos(\frac{3}{2}x^2)$ $-x^{-1} - \cos(\frac{3}{2}x^2)$ $\frac{1}{3}x^{-3} + 3\cos(3x)$

Which of the following formulas is equivalent to $\sum_{i=1}^{n}(5 + 2i)$?

$6n + n^2$ $\frac{(5+2n)(6+2n)}{2}$ $5n + 2n^2$ $5 + 2n$ $5 + 10n + 4n^2$

If $F(x) = \int_{-3}^{x} \sqrt{4 + \sin t}\, dt$, what is $F'(0)$? (Hint: first find $F'(x)$.)

2 1 3 0 4

Evaluate: $\int_0^4 \sqrt{16 - x^2}\, dx$. Hint: think geometrically!

4π $\frac{2}{3}$ $\frac{128}{3}$ 16π 4