Brief Article

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Math 120: Calculus	Name:________________________	**Exam I**	Tutorial
Instructor:________________________	*February 13, 1996*		Tutorial
Section:________________________			

Calculators are not allowed. Hand in this answer page only. Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 19 multiple choice questions, worth 5 points each. An additional 5 points will be given for your correct tutorial section number.

You are taking this exam under the honor code.

Let A be the region in the plane bounded above by the curve $y = 2 - x^2$, below by the curve $y = x^2$ and to the left by the y-axis. (This region is given in the diagram below.) Find the volume of the solid obtained by revolving this region about the y-axis. You don't have to find the actual volume–just express it in terms of integrals.

$$\pi \int_0^1 (\sqrt{y})^2 \, dy + \pi \int_1^2 \left(\sqrt{2-y}\right)^2 dy$$

$$\pi \int_0^1 \left(2 - x^2\right)^2 dx - \pi \int_0^1 \left(\sqrt{x^2}\right)^2 dx$$

$$\pi \int_0^1 \left(x^2\right)^2 dx + \pi \int_1^2 \left(2 - x^2\right)^2 dx$$

$$\pi \int_0^{\sqrt{2}} \left(2 - x^2\right)^2 dx - \pi \int_0^{\sqrt{2}} \left(x^2\right)^2 dy$$

$$\pi \int_0^{2} \left(\sqrt{2 - y}\right)^2 dy - \pi \int_0^{1} (\sqrt{y})^2 dy$$

Let $F(x) = \int_{\pi/2}^{x} \sin^2 t \, dt$. Find $F'(x)$. $\sin^2 x$ $2 \sin x \cos x$ $-2 \sin x \cos x$ $\sin^2 x - 1$ $2 \sin x \cos x - 1$

Let $F(x) = \int_{2}^{x^3} t^2 \, dt$. Find $F'(x)$.

$3x^8$ x^6 $2x^3$ $6x^5$ $3x^7$

Let $F(x) = \int_{x^2}^{x^3} \sin t \, dt$. Find $F'(x)$.

$3x^2 \sin(x^3) - 2x \sin(x^2)$

$3x^2 \sin(x^3) + 2x \sin(x^2)$

$\sin(x^3) - \sin(x^2)$

$-\cos(x^3) + \cos(x^2)$

$\cos(x^3) - \cos(x^2)$

Let B be the region bounded above by the line $y = 2$, below by the curve $y = x^2$ and to the left by the y-axis. (This region is given in the diagram below.) Find the volume of the solid obtained by revolving this region about the line $y = 2$. You don't have to find the actual volume– just express it in terms of integrals.

$$\pi \int_0^{\sqrt{2}} (2 - x^2)^2 dx$$

$$\pi \int_0^{\sqrt{2}} (x^2)^2 dx$$

$\pi \int_0^2 (\sqrt{y})^2 dy$

$\pi \int_0^2 (2 - x^2)^2 dx$

$\pi \int_0^2 (2 - \sqrt{y})^2 dy$

Evaluate $\int_{-2}^{1} \sqrt{x+3}\ dx$.

$\frac{14}{3}$

7

$\frac{2}{3}$

$\frac{16}{3}$

This integral cannot be evaluated.

Evaluate the following indefinite integral: $\int \frac{x}{\sqrt{1+x^2}}\ dx$

$\sqrt{x^2+1} + C$

$2\sqrt{x^2+1} + C$

$\frac{1}{2}\sqrt{x^2+1} + C$

$\frac{1}{2\sqrt{x^2+1}} + C$

$\frac{1}{\sqrt{x^2+1}} + C$

Evaluate the following indefinite integral: $\int \sec^2 3\theta\ d\theta$

$\frac{1}{3}\tan 3\theta + C$

$\frac{1}{9}\sec^3 3\theta + C$

$\frac{1}{9}\tan^3 3\theta + C$

$3\tan 3\theta + C$

$\frac{1}{3}\sec^3 3\theta + C$

The velocity function (in meters per second) for a particle moving along a line is given by $v(t) = t^2 - 2t$. Find the *displacement* during the time interval $0 \le t \le 3$.

0 m

3 m

$\frac{8}{3}$ m

4 m

-6 m

A tank has the shape illustrated in the diagram below. The sides are isosceles triangles, and a magnified side view is also provided. It is filled with water to a height of 1 m. (See picture.) Which of the following integrals represents the work required to empty the tank by pumping all of the water to the top of the tank? (The density of water is 1000 kg/m^3 and the acceleration due to gravity is 9.8 m/sec^2. Notice that the answers don't all have the same limits of integration.)

$$\int_2^3 9800 \cdot 10 \left(\frac{2(3-x)}{3}\right) x dx$$

$$\int_0^2 9800 \cdot 10 \left(\frac{2(1-x)}{3}\right) x dx$$

$$\int_2^3 9800 \cdot 10 \left(\frac{2}{3}\right) x dx$$

$$\int_2^3 9800 \cdot 10 \left(\frac{2(1-x)}{3}\right)(2+x)dx$$

$$\int_0^2 9800 \cdot 10 \left(\frac{2(3-x)}{3}\right) x dx$$

Suppose that 16 J of work are needed to stretch a spring from its natural length of 10 cm to a length of 14 cm. How much work is needed to stretch it from 12 cm to 15 cm?

21 J

$\frac{27}{2}$ J

$\frac{21}{4}$ J

$\frac{81}{4}$ J

11 J

Find the average value of the function $f(x) = x^3$ on the interval $[1, 3]$.

10

20

$\frac{20}{3}$

80

40

Find the average value of the function $f(x) = \cos^2 x \sin x$ on the interval $[0, \pi]$.

$\frac{2}{3\pi}$

0

$\frac{2}{3}$

$-\frac{1}{3\pi}$

$\frac{1}{3\pi}$

The velocity function (in meters per second) for a particle moving along a line is given by $v(t) = t^2 - 2t$. Find the *total distance travelled* during the time interval $0 \leq t \leq 3$.

$\frac{8}{3}$ m

3 m

0 m

4 m

6 m

Find the volume of the solid obtained by rotating the following region about the x-axis. The region is bounded by the curves $y = \sqrt{2x-1}$, the x-axis, the line $x = 1$ and the line $x = 5$.

20π

$\frac{26\pi}{3}$

$\frac{13}{6}$

$\frac{\pi}{3}\left[5^{2/3} - 1\right]$

$\frac{\pi}{6}\left[5^{2/3} - 1\right]$

A cable weighing 4 lbs/ft is used to lift a 500 lb piano 200 ft up the side of a building. Which of the following represents the amount of work done?

$\int_0^{200} (4x + 500)dx$

$500 + \int_0^{200} 4x dx$

$\int_0^{200} [4(200 - x) + 500]dx$

$500 + \int_0^{200} 4(200 - x)dx$

$\int_0^{200} 504x dx$

A certain experiment involves heating a metal rod, which is 3 m long. It is discovered that the temperature of the rod (in °C), at a distance of x meters from one end, is $4x + 100$. What is the average temperature of the rod?

106°

318°

102°

112°

104°

Evaluate the following indefinite integral: $\int \frac{\sin\sqrt{x}}{\sqrt{x}} dx$

$-2\cos\sqrt{x} + C$

$2\cos\sqrt{x} + C$

$-\frac{1}{2}\cos\sqrt{x} + C$

$\frac{1}{2}\cos\sqrt{x} + C$

$\cos\sqrt{x} + C$

An animal population is increasing at a rate of $100 + 20t$ per year, where t is measured in years. (Note that this formula represents the rate, not the population itself.) By how much does the population increase between the second and fifth years?

510

60

210

750

240