Math 119 Name:________________

Final exam Instructor:________________

May 8, 1997 Section:________________

Find $\lim_{h\to 0} \frac{\cos(\frac{\pi}{2}+h)}{h}$ -1 0 π $-\pi$ -2

Find all the horizontal and vertical asymptotes of $y = \frac{x+1}{\sqrt{x}-1}$. No horizontal asymptote; $x = 1$ $y = 1$; $x = 1$ $y = -1$; $x = -1$ $y = 0$; $x = -1$ $y = -1$; no vertical asymptote

A Norman window is constructed from a rectangular sheet of glass surmounted by a semicircular sheet of glass. The light that enters through a window is propotional to the area of the window. What is the dimension of the semicircle's **diameter** of the Norman window having a perimeter of 30 feet which admits the most light? $\frac{60}{4+\pi}$ $\frac{30}{4+\pi}$ $\frac{15}{2+\pi}$ 20 15

Let $F(x) = \int_0^x \sqrt{\frac{u-1}{u+1}}\, du$. Calculate $F'(1)$. 0 1 -1 $\frac{1}{2}$ $-\frac{1}{2}$

Evaluate the integral $\int_{-1}^{1}(1-|x|)\,dx$ 1 -1 $\frac{1}{2}$ $-\frac{1}{2}$ 0

Where is the graph of the function

$$f(x) \;=\; x^3 - 6x^2 + 12x - 5$$

concave up, and where is it concave down? concave up on $(2,\infty)$, concave down on $(-\infty,2)$ concave up on $(-\infty,0)$, concave down on $(0,\infty)$ concave up on $(-\infty,0)$ and $(2,\infty)$, concave down on $(0,2)$ concave up on $(0,2)$, concave down on $(-\infty,0)$ and $(2,\infty)$ concave up on $(-\infty,2)$, concave down on $(2,\infty)$

$$f(x) = \begin{cases} 1 & \text{if } x \le 0 \\ \cos x & \text{if } 0 < x < \pi/2 \\ -1 & \text{if } x \ge \pi/2 \end{cases}$$

Where is f continuous? Everywhere but at $x = \pi/2$ Everywhere but at $x = 0$ Everywhere but at $x = 0$ and $x = \pi/2$ Everywhere Nowhere

If $f(x) = \sqrt{2x+3}$, which of the following limits conforms to the definition of $f'(1)$? $\lim_{h\to 0} \frac{\sqrt{5+2h}-\sqrt{5}}{h}$ $\lim_{h\to 0} \frac{\sqrt{2h+3}-\sqrt{5}}{h}$ $\lim_{h\to 0} \frac{\sqrt{5+h}-\sqrt{5}}{h}$ $\lim_{h\to 0} \frac{\sqrt{3}+\sqrt{h}-\sqrt{5}}{h}$ $\lim_{h\to 0} \frac{\sqrt{2h}+\sqrt{3}-\sqrt{5}}{h}$

Suppose f and g are functions satisfying the following conditions:

$f(0) = 1$	$f(1) = 0$	$g(0) = 3$	$g(1) = -2$
$f'(0) = 2/3$	$f'(1) = -1/2$	$g'(0) = -5$	$g'(1) = 1/3$

If $h(x) = g\big(f(x)\big)$, what is $h'(1)$? $5/2$ $-1/6$ $2/9$ $-10/3$ 0

Find the slope of the tangent to the graph of the equation

$$x^2 + y^2 + xy = 1$$

at $(-1,0)$. -2 1/2 1 2 does not exist

Suppose a tank holds 5000 gallons of water, and it takes 40 minutes to drain a full tank. According to Toricelli's law, the volume V of water remaining in the tank at the end of t minutes is given by the formula

$$V \;=\; 5000\Big(1 - \frac{t}{40}\Big)^2 \quad \text{for} \quad 0 \le t \le 40$$

How fast is the tank **draining** at the end of 20 minutes? (All answers are in gallons per minute.) 125 5000 1250 250 500

Find the absolute maximum and minimum values of the function

$$f(x) = \frac{x}{4} + \frac{9}{x}$$

on the interval $[1,8]$ maximum value $= 37/4$, minimum value $= 3$ maximum value $= 37/4$, minimum value $= 25/8$ maximum value $= 12$, minimum value $= 25/8$ maximum value $= 39/4$, minimum value $= 13/4$ maximum value $= 12$, minimum value$= 3$

If $f(0) = 4$ and $f'(0) = 2$, find the slope of the graph

$$y = \sqrt{x + f(x)}$$

at the point $(0,2)$ $3/4$ $1/4$ $\sqrt{3}/2$ $1/\sqrt{2}$ $\sqrt{2}$

Letting $f(x) = \frac{x-1}{x+1}$, find $\frac{d}{dx}\int_1^{3x} f$ $\frac{3(3x-1)}{3x+1}$ $\frac{3x-1}{3x+1}$ $\frac{x-1}{x+1} - 1$ $-\frac{x-1}{x+1}$ $\frac{3(3x-1)}{3x+1} - 1$

For what x is the line tangent to the graph

$$y = x^2 + x + 1$$

parallel to the line through the points $(0,3)$ and $(-1,1)$. $1/2$ $-3/2$ -2 3 1

The volume of a sphere of radius r is given by the formula

$$V = \frac{4}{3}\pi r^3$$

When helium is pumped into a spherical balloon, both the radius and the volume change with respect to time. Suppose that at a certain moment the volume is increasing at the rate of 8π cu. ft. per minute, and the radius is increasing at the rate of $1/2$ ft per minute. What is the volume of the balloon at that moment? $\frac{32\pi}{3}$ $\frac{\pi}{4}$ 16π $\frac{\pi}{16}$ $\frac{4\pi}{3\sqrt{2}}$

Find $\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2}$, if it exists 4 2 1 0 does not exist

Suppose $f(x)$ is defined and has a derivative for all x in some interval $a < x < b$, and for some point c in the interval $f'(c) = 0$ and $f''(c) = 0$. Which of the following statements is true? (Only one of them is true.) The function can not have either a local maximum or minimum at $x = c$ The function must have a local minimum at $x = c$ The function must have a local maximum at $x = c$ The function must have an inflection point at $x = c$ No conclusion can be drawn about whether there is an inflection point or local extremum without further information

A box with a square base and a top is to be built for a cost of \$60. The material for the base costs \$3 per sq. ft.; the material for the top and sides costs \$2 per sq. ft. What is the maximum volume of such a box? (Answers are in cubic feet.) 10 24 16 20 there is no maximum

How many real roots does the cubic equation $x^3 + 1$ have? 1 none 2 3 4

Find all the critical points of the function

$$f(x) = \sin x + \cos x$$

in the interval $(0, 2\pi)$. $\pi/4$ and $5\pi/4$ π $3\pi/4$ and $7\pi/4$ $\pi/2$, π and $3\pi/2$ $\pi/2$ and $3\pi/2$

Which statement is false for the curve $f(x) = \frac{x}{1+x}$? its graph is symmetric around the origin its critical number is -1 it does not have an inflection point it is continuous everywhere except at $x = -1$ it is concave up on $(-1, \infty)$

Suppose f is a differentiable function such that $f(g(x)) = x$ and $f'(x) = 1 + [f(x)]^2$. What is $g'(x)$? $\frac{1}{1+x^2}$ $\frac{1}{1+x}$ $-\frac{1}{1+x^2}$ $\frac{1}{x}$ $-\frac{1}{x}$

Find $f(x)$ if $f'(x) = \sec^2 x$ and $f(0) = 0$. $\tan x$ $\tan x + 1$ $\tan x - 1$ $\sec x$ $\sec x - 1$

Which of the following is equivalent to

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\sqrt{1 + (\frac{3i}{n})^2}$$

? $\int_0^3 \sqrt{1+x^2}\,dx$ $\int_0^1 \sqrt{1+x^2}\,dx$ $\int_0^3 \sqrt{x^2}\,dx$ $\int_0^3 \sqrt{1+3x^2}\,dx$ $\int_0^1 \sqrt{1+3x^2}\,dx$

Which of the following statements is false? If f and g are integrable on $[a,b]$, then $\int_a^b [f(x)g(x)]\,dx = \left(\int_a^b f(x)\,dx\right)\left(\int_a^b g(x)\,dx\right)$ If f is differentiable at a, then f is continuous at a All continuous fuctions have antiderivatives If p is a polynomial and b is a real number, then $\lim_{x\to b} p(x) = p(b)$ If $f'(a)$ exists, then $\lim_{x\to a} f(x) = f(a)$

Find $\int \cos^4 x \, \sin x \, dx$ $-\frac{1}{5}\cos^5 x + C$ $\frac{1}{5}\cos^5 x + C$ $-\frac{1}{4}\cos^4 x + C$ $\frac{1}{5}\sin^5 x + C$ $-\frac{1}{4}\sin^5 x + C$

Which of the following statements is true? If f and g are integrable on $[a, b]$, then $\int_a^b [f(x) + g(x)]\, dx = \int_a^b f(x)\, dx + \int_a^b g(x)\, dx$ $D(\sec x) = D(\tan^2 x)$ If f and g are differentiable and $f(x) \geq g(x)$ for $a \leq x \leq b$, then $f'(x) \geq g'(x)$ for $a < x < b$. $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$ If f has an absolute minimum value at c, then $f'(c) = 0$

Which of the following is true? if $f(-x) = -f(x)$, then f is symmetric around the origin if f has an absolute minimum value at c, then $f'(c) = 0$ if $f''(c) = 0$, then $(c, f(c))$ is an inflection point of the curve $y = f(x)$ if $f'(x) = g'(x)$ for $0 < x < 1$, then $f(x) = g(x)$ for $0 < x < 1$. $\sum_{i=1}^n a_i b_i = \left(\sum_{i=1}^n a_i\right)\left(\sum_{i=1}^n b_i\right)$

There is a unique number that falls between the lower and upper sums of the function

$$f(x) = x^2 - 2\,, \quad 1 \leq x \leq 4$$

for every partition of the interval $[1, 4]$. What is that number? 15 21 -15 1 0

Which of the following sums is the Riemann sum for the function

$$f(x) = \frac{1}{1+x}\,, \quad 0 \leq x \leq 2$$

if the interval $[0, 2]$ is partitioned into four equal segments and c_i is the midpoint of the i^{th} interval? $\frac{2}{5} + \frac{2}{7} + \frac{2}{9} + \frac{2}{11}$ $\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}$ $\frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7}$ $\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8}$ $\frac{2}{5} + \frac{2}{6} + \frac{2}{7} + \frac{2}{8}$

State the Fundamental Theorem of Calculus (two parts).

That's all folks!