Math 119
Final Exam
December 16, 1998

1. Find the domain of $f(x) = \left(\dfrac{(x-1)^2}{x-2}\right)^{1/3}$.

(a) $(1, 2)$
(b) $(1, \infty)$
(c) $[2, \infty)$
(d) $(-\infty, 2) \cup (2, \infty)$
(e) $(-\infty, 1) \cup (2, \infty)$

2. Find $\lim\limits_{x \to 0} \dfrac{\sqrt{x^2+3} - \sqrt{3}}{x}$.

(a) $\dfrac{1}{\sqrt{3}}$
(b) $\dfrac{1}{3\sqrt{3}}$
(c) $\sqrt{3}$
(d) $\dfrac{1}{\sqrt{6}}$
(e) 0

3. Find $\lim\limits_{x \to -1} \dfrac{x+1}{x^2 - x - 2}$.

(a) $-\dfrac{1}{3}$
(b) $-\dfrac{1}{2}$
(c) -1
(d) $-\dfrac{2}{3}$
(e) $-\dfrac{3}{2}$

The next 2 problems concern the following graph of the function $f(x)$.

4. Which of the following is closest to the value of $f'(13)$?

(a) 2.5
(b) 0.9
(c) 0.4
(d) -2.5
(e) The derivative doesn't exist.

5. Only one of the following assertions is true; find the true statement.

(a) $f'(x)$ is non-negative for $0 \leq x \leq 13$.
(b) $f(12)$ is larger than $f(13)$.
(c) $f'(10)$ is smaller than $f'(13)$.
(d) $f'(15)$ is larger than $f'(13)$.
(e) f' is an increasing function on the interval $[0, 13]$.

6. Find the coordinates of the point on the curve

$$y = x^2 + x + 1$$

where the tangent line is parallel to the line $y = 5x + 2$.

(a) $(1, 3)$
(b) $(2, 7)$
(c) $(0, 1)$
(d) $(-1, 1)$
(e) $(-2, 3)$

7. Let $f(x) = 2x^2 + x + 3$ and let $h \neq 0$. Which of the following represents the quantity:

$$\frac{f(x+h) - f(x)}{h}$$

Note: This is not a limit.

(a) $4x + 2h + h^2$

(b) $4x + 1$

(c) $4x + 2h + 1$

(d) $2x^2 + x - 2h$

(e) $4x + 2h^3 + 1$

8. A stone is dropped into a lake, creating a circular ripple with radius increasing at a rate of 20 in/sec. Find the rate at which the area within the circle is increasing when the radius is 3 in.

(a) 6π

(b) 18π

(c) 60π

(d) 100π

(e) 120π

9. Find $\frac{d}{dx}\left[\sin(x^2+3)\right]$.

(a) $\cos(2x)$

(b) $2x\cos(x^2+3)$

(c) $\cos(2x^3+6x)$

(d) $-\cos(x^2+3)$

(e) $\cos(x^2+3)+2x$

10. Find $\frac{d}{dx}\left[\frac{x+1}{(x^2+3)^2}\right]$.

(a) $\frac{-3x^2-2x+1}{(x^2+3)^3}$

(b) $\frac{5x^2-2x+1}{(x^2+3)^3}$

(c) $\frac{-x^3-x^2+x-3}{(x^2+3)^3}$

(d) $\frac{1}{4x(x^2+3)}$

(e) $\frac{3-4x-3x^2}{(x^2+3)^3}$

11. Find the absolute maximum and absolute minimum values of the function

$$f(x) = x^4 - 4x + 8$$

on the interval $[0, 2]$.

(a) maximum value 16, minimum value 5

(b) maximum value 16, minimum value 1

(c) maximum value 2, minimum value 0

(d) maximum value 8, minimum value 5

(e) maximum value 16, minimum value 8

12. If $f(x) = x\sin(2x)$, find the second derivative $f''(x)$.

(a) $f''(x) = \cos(2x)$

(b) $f''(x) = 4\cos(2x)$

(c) $f''(x) = 4\cos(2x) - 4x\sin(2x)$

(d) $f''(x) = 3\cos x - 4x\sin x$

(e) $f''(x) = 2\cos(2x) + 4x\sin(2x)$

13. Let $x^2 + y^2\sin x = 2$. What is $\dfrac{dy}{dx}$?

(a) $\dfrac{dy}{dx} = -\dfrac{x}{y\sin x}$

(b) $\dfrac{dy}{dx} = 2 - \dfrac{y^2\cos x + 2x}{2y\sin x}$

(c) $\dfrac{dy}{dx} = 2x + 2y\sin x$

(d) $\dfrac{dy}{dx} = -\dfrac{y^2\cos x + 2x}{2y\sin x}$

(e) $\dfrac{dy}{dx} = \dfrac{2 - x^2}{y\sin x}$

14. Suppose that for $x \in [0, 2\pi]$, we define $g(x) = \sin^2 x$. On what intervals is g increasing?

(a) $[0, 2\pi]$

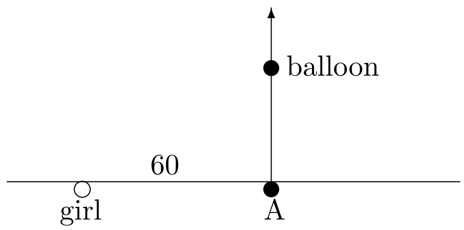
(b) $\left[0, \frac{\pi}{2}\right] \cup \left[\pi, \frac{3\pi}{2}\right]$

(c) $\left[\frac{\pi}{2}, \pi\right] \cup \left[\frac{3\pi}{2}, 2\pi\right]$

(d) $[0, \pi]$

(e) $[\pi, 2\pi]$

15. A balloon is rising vertically from a point A. It has constant speed of 5m/sec. A girl is watching from a position exactly 60m from A. How fast is the distance between the girl and the balloon increasing when the balloon is 80m high?



(a) 2m/sec.
(b) 4m/sec.
(c) 6m/sec.
(d) 8m/sec.
(e) 10m/sec.

16. On what intervals is $f(x) = \dfrac{3x-2}{4x-3}$ concave up?

(a) $\left(-\infty, \frac{3}{4}\right)$
(b) $\left(\frac{3}{4}, \infty\right)$
(c) none
(d) $\left(-\infty, \frac{3}{4}\right) \cup \left(\frac{3}{4}, \infty\right)$
(e) $(-\infty, \infty)$

17. Compute $\lim_{x \to \infty} \dfrac{3x^3}{\sqrt{3x^3 + 1}}$

(a) $-\infty$
(b) 0
(c) 1
(d) $\sqrt{3}$
(e) ∞

18. Consider $f(x) = \dfrac{4}{x} + x$. Then f has

(a) local minima at $x = 2$ and $x = -2$.
(b) local maxima at $x = 2$ and $x = -2$.
(c) a local minimum at $x = -2$ and a local maximum at $x = 2$.
(d) a local minimum at $x = 2$ and a local maximum at $x = -2$.
(e) no local extrema.

19. The product of two positive numbers is 400. What is the minimum their **sum** can be?

(a) -20
(b) 10
(c) 40
(d) 50
(e) 100

20. $\displaystyle\sum_{i=1}^{100}(3i+1) =$

(a) 15150
(b) 15250
(c) 100200
(d) 151250
(e) 300100

21. The graph below is the graph of the **derivative** of a function $f(x)$.

Which of the following graphs best represents the graph of $f(x)$?

(a) **(b)** **(c)**
(d) **(e)**

22. Find the area under the graph of $y = x^3 + 1$ above the interval from 0 to 2 on the x-axis. The region is shown in the picture below, which is not to scale.

(a) 1

(b) 4.5

(c) 6

(d) 7

(e) 9

23. Consider the function $f(x) = x^2 + 1$ on the interval $[1, 2]$. Use the partition $\{1, \frac{3}{2}, 2\}$. Take x_i^* to be the right endpoint of the subinterval $[x_{i-1}, x_i]$. Calculate the Riemann sum $\sum_{i=1}^{n} f(x_i^*)\Delta x_i$ where $\Delta x_i = x_i - x_{i-1}$.

(a) $\frac{33}{8}$

(b) $\frac{23}{4}$

(c) $\frac{21}{8}$

(d) $\frac{10}{3}$

(e) 4

24. Calculate $\int (x^2 - 5x)\, dx$.

(a) $x - 5$

(b) $-\frac{13}{6}$

(c) $\frac{x^3}{3} - \frac{5}{2}x^2 + C$

(d) $x^2 - 5x + C$

(e) $\frac{x^3}{2} - 5x^2 + C$

25. Calculate $\int_{-1}^{2} (2x - 1)\, dx$.

(a) 4

(b) $\frac{3}{2}$

(c) 0

(d) 6

(e) 2