

Exam I

Tutorial Instructor: _____

February 21, 1995

Tutorial Section: _____

Calculators are not allowed. Hand in this answer page only. Record your answers to the multiple choice problems by placing an \times through one letter for each problem on this answer sheet. There are 19 multiple choice questions, worth 5 points each. An additional 5 points will be given for your correct tutorial section number.

You are taking this exam under the honor code.

Let $f(x) = \int_2^{3x^2} \frac{1}{1+t^2} dt$. What is $f'(x)$? $\frac{6x}{1+9x^4}$ $\frac{3x^2}{1+9x^4}$ $\frac{x^3}{1+x^2}$ $\frac{3x^2}{1+x^2}$ $\frac{x^3}{1+9x^4}$

Find the following antiderivative: $\int x^3 \sqrt{1+x^4} dx$ $\frac{1}{6}(1+x^4)^{3/2} + C$ $\frac{8}{3}(1+x^4)^{3/2} + C$
 $\frac{1}{4}x^4 + \frac{1}{6}x^6 + C$ $\pm \frac{1}{4}x^4 \pm \frac{1}{6}x^6 + C$ $\frac{1}{6}x^4(1+x^4)^{3/2} + C$

Find the following antiderivative: $\int \frac{\sin x}{\cos^2 x} dx$ $\frac{1}{\cos x}$ $-\frac{1}{\cos x}$ $\frac{1}{2} \sin^2 x \sec^3 x - \ln(\cos x)$

Find the volume of the solid obtained by rotating the region bounded by the curves $y = 2x$, $x = 0$, $x = 3$ and $y = 0$ about the x -axis. 36π 18π 9π $18\pi^2$ 4π

Consider the region bounded above by the curve $y = x^2$, below by $y = 0$ and to the left and right respectively by $x = 0$ and $x = 2$. Which of the following integrals represents the volume of the solid obtained by rotating this region about the line $x = 2$? $\pi \int_0^4 (2 - \sqrt{y})^2 dy$ $\pi \int_0^4 [(2)^2 - (\sqrt{y})^2] dy$ $\pi \int_0^2 (2 - x^2)^2 dx$ $\pi \int_0^4 (\sqrt{y})^2 dy$
 $\pi \int_0^2 [(4)^2 - (x^2)^2] dx$

The natural length of a certain spring is 10 cm. The work required to stretch it from its natural length to a length of 12 cm is 6 joules. How much work is required to stretch it from 12 cm to 14 cm? 18 joules 6 joules 78 joules $\frac{78}{11}$ joules 24 joules

Evaluate the following definite integral: $\int_{-1}^1 x(x^2 - 3)^6 dx$. 0 $\frac{1}{7}$ 128 -386 6

If $y = \frac{\sqrt{x^2+3}}{e^x \cos^3 x}$ find $\frac{dy}{dx}$ using logarithmic differentiation. $\frac{dy}{dx} = y \cdot \left[\frac{x}{x^2+3} - 1 + \frac{3 \sin x}{\cos x} \right]$
 $\frac{dy}{dx} = y \cdot \left[\frac{x}{x^2+3} - x + \frac{3 \sin x}{\cos x} \right]$ $\frac{dy}{dx} = y \cdot \left[\frac{x}{x^2+3} + \frac{3 \sin x}{\cos x} \right]$ $\frac{dy}{dx} = y \cdot \left[\frac{2x}{x^2+3} - x - \frac{3 \sin x}{\cos x} \right]$
 $\frac{dy}{dx} = y \cdot \left[\frac{2x}{x^2+3} - 1 - \frac{3 \sin x}{\cos x} \right]$

If $f(x) = e^{x^2}$ find $f'(3)$. $6e^9$ $2e^6$ $2e^9$ e^9 $6e^6$

A tank has the shape of an inverted circular cone with height 9 m and base radius 3 m. It is filled with water to a height of 5 m. (See picture.) Which of the following integrals represents the work required to empty the tank by pumping all of the water to the top of

the tank? (The density of water is 1000 kg/m^3 and the acceleration due to gravity is 9.8 m/sec^2 . Notice that the answers don't all have the same limits of integration.)

$$\int_0^5 9800 \left[\pi \left(\frac{y}{3} \right)^2 \right] (9 - y) dy \quad \int_0^9 9800 \left[\pi (3y)^2 \right] (9 - y) dy \quad \int_0^5 9800 \left[\pi (3y)^2 \right] (9 - y) dy$$

$$\int_5^9 9800 \left[\pi \left(\frac{y}{3} \right)^2 \right] (5 - y) dy \quad \int_5^9 9800 \left[\pi (3y)^2 \right] (5 - y) dy$$

Find the average value of the function $f(x) = x^2 - 1$ over the interval $[-3, 3]$. 2 12 0 3 1

Simplify (the variable z represents a positive real number): $\frac{z^2 z^{1/2}}{z^{-2}} z^{9/2} z^3 z^{-1} z^{1/2} z^2$

What is the domain of the function $e^x \ln(1 - x^2)$? $(-1, 1)$ $(-\infty, -1) \cup (1, \infty)$ $[-1, 1]$ $[0, 1]$ $(0, 1)$

Which of the following is equal to $e^{\frac{1}{2} \ln x}$? \sqrt{x} $\frac{x}{2}$ $\left(\frac{1}{2}\right)^x$ $\frac{1}{x^2}$ $e^{\sqrt{x}}$

Solve for x : $2 \log_9(x) = 1$. $x = 3$ $x = \pm 3$ $x = 9$ $x = 81$ $x = \pm 81$

Let $f(x) = e^x + e^{-x}$. Where is $f(x)$ increasing and where is it concave up? increasing on $[0, \infty]$; concave up everywhere increasing on $[-\infty, 0]$; concave up everywhere increasing everywhere; concave up everywhere increasing everywhere; concave up on $[0, \infty]$ increasing on $[0, \infty]$; concave up on $[0, \infty]$

Find $\int_0^1 \frac{1}{2x+1} dx$ $\ln(\sqrt{3})$ $\frac{1}{2} \ln 3$ $\ln \frac{1}{3}$ $\frac{2}{9}$

Find the derivative of the function $f(x) = \ln(\ln(3x))$. $\frac{1}{x \ln(3x)}$ $\frac{1}{3x \ln(3x)}$ $\frac{3}{x \ln(3x)}$

$$\frac{1}{x \ln x} \quad \frac{3}{x \ln x}$$

Find $\int e^x (1 + e^x)^8 dx$ $\frac{1}{9} (1 + e^x)^9 + C$ $(1 + e^x)^9 + C$ $\frac{1}{8} (1 + e^x)^8 + C$ $e^x + e^{9x} + C$ $\frac{1}{9} e^x (1 + e^x)^9 + C$