Math 120: Calculus Name:________________

Exam II Tutorial Instructor:________________

March 28, 1995 Tutorial Section:________________

Calculators *are* allowed on this exam. Hand in this answer page only. Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 19 multiple choice questions, worth 5 points each. An additional 5 points will be given for your correct tutorial section number.

You are taking this exam under the honor code.

Let $f(x) = x^{(x^2)}$. What is $f'(x)$? $x^{(x^2)}\left[x + 2x\ln x\right]$ $x^{2x}\left[2 + 2\ln x\right]$ $x^2 \cdot x^{(x^2-1)}$ $x^{(x^2)} \cdot 2x$ $x^{(x^2)}\left[2 + 2\ln x\right]$

Let $f(x) = x^x$. What is $f'(x)$? $x^x\left[1 + \ln x\right]$ $x \cdot x^{x-1}$ $x^x \cdot \left[1 + \frac{1}{x}\right]$ $x^x \cdot \frac{1}{x}$ $xe^x + e^x$

Fred invests \$5000 in an account at an annual interest rate of 10%, compounded continuously. How much will he have (in dollars) in the account after 20 years? $5000e^2$ $500e^{20}$ 10,000 15,000 $5000\left(1 + .10\right)^{20}$

A certain radioactive substance is being studied. It is observed that after 100 days, 75% of the original amount remains. What is the half-life of that radioactive substance (in days)? $\dfrac{100\ln(.5)}{\ln(.75)} \approx 241$ $200\,\dfrac{200\ln(.75)}{\ln(.5)} \approx 83$ $1000 \cdot |\ln(.75)| \approx 288$ $100 \cdot (\ln(.75) - \ln(.5)) \approx 405$

A bacteria culture starts with 600 bacteria and after 4 hours there are 6000 bacteria. Find an expression for the number of bacteria after t hours. $600 \cdot 10^{t/4}$ $600 \cdot e^{t/4}$ $(0.6) \cdot 10^t$ $600 \cdot 10^t$ $600e^t$

Find the following antiderivative: $\displaystyle\int \frac{e^x}{1 - e^{2x}}dx$. $\left(\text{Hint: Find a substitution, then use the fact that } \dfrac{1}{1-u^2} = \dfrac{1/2}{1-u} + \dfrac{1/2}{1+u}\right)$ $\frac{1}{2}\ln\left|\frac{1+e^x}{1-e^x}\right| + C$ $\frac{1}{2}\ln\left|1 - e^{2x}\right| + C$ $\frac{1}{2}\ln\left|\frac{1-e^x}{1+e^x}\right| + C$ $e^x + e^{-x} + C$ $-\ln\left|1 - e^{2x}\right| + C$

Evaluate the following definite integral: $\displaystyle\int_0^{\pi} x \sin x \, dx$ π $4\pi - 2$ $2\pi - 2$ $4\pi - 4$ $\pi - 2$

Evaluate the following definite integral: $\displaystyle\int_0^{\frac{\pi}{2}} \sin^2 x \cos x \, dx$ $\frac{1}{3}$ $\frac{\pi^2}{4}$ $\frac{1}{3} \cdot \frac{\pi^3}{8}$ 0 $-\frac{1}{3}$

In the partial fraction decomposition

$$\frac{x^2+1}{x^3-x} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$$

find the value of A. (You don't have to find the values of B or C.) -1 1 $1/2$ $-1/2$ 2

Use long division to divide $\dfrac{x^3+2x^2+2x+1}{x^2+1}$ $x+2+\dfrac{x-1}{x^2+1}$ $x+1+\dfrac{2x}{x^2+1}$ $x+1+\dfrac{x+1}{x^2+1}$ $x+2+\dfrac{2x+1}{x^2+1}$ $x+2+\dfrac{3x-1}{x^2+1}$

It is a fact, which you don't have to verify, that

$$\frac{x^2+1}{x(2x-1)^2} = \frac{1}{x} - \frac{3/2}{2x-1} + \frac{5/2}{(2x-1)^2}$$

Using this, find $\int \frac{x^2+1}{x(2x-1)^2} dx$ $\ln|x| - \frac{3}{4}\ln|2x-1| - \frac{5/4}{2x-1} + C$ $\ln|x| - 3\ln|2x-1| - \frac{5}{2x-1} + C$ $\ln|x| + 3\ln|2x-1| + \frac{5}{2x-1} + C$ $\ln|x| + \frac{3}{4}\ln|2x-1| + \frac{5/4}{2x-1} + C$ $\ln|x| - \frac{3}{4}\ln|2x-1| - \frac{5}{4}\ln\left((2x-1)^2\right) + C$

Evaluate the following definite integral: $\int_0^{\pi} \cos^2 2x\, dx$ $\frac{\pi}{2}$ π 0 $\frac{\pi}{4}$ 2π

Evaluate the following definite integral: $\int_0^{\frac{\pi}{2}} \cos^3 x\, dx$ $2/3$ $\frac{\pi}{2} - \frac{\pi^3}{24}$ $\frac{\pi}{2}$ 0 1

Evaluate the following definite integral: $\int_1^2 x\ln x\, dx$. (Hint: $\ln 1 = 0$.) $2\ln 2 - \frac{3}{4}$ $2\ln 2 - \frac{3}{4} - \frac{e}{2}$ $2\ln 2$ $2\ln 2 - 1$ $2\ln 2 - e - 1$

Evaluate the following definite integral: $\int_0^1 \frac{dx}{\sqrt{4-x^2}}$ $\pi/6$ $\pi/3$ $\pi/4$ π 0

In order to integrate

$$\int \frac{x^2-10}{x(x+1)(3x-1)^2} dx$$

by partial fractions, which of the following is the correct decomposition to set it up? $\frac{x^2-10}{x(x+1)(3x-1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C_1}{3x-1} + \frac{C_2}{(3x-1)^2}$ $\frac{1}{x(x+1)(3x-1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C_1}{3x-1} + \frac{C_2}{3x-1}$ $\frac{x^2-10}{x(x+1)(3x-1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C_1}{3x-1} + \frac{C_2}{3x-1}$ $\frac{x^2-10}{x(x+1)(3x-1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(3x-1)^2}$ $\frac{1}{x(x+1)(3x-1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(3x-1)^2}$

Suppose that y is some quantity changing over time t, and that the rate at which it changes
is proportional to the quantity present at any given time. In other words, suppose that $\frac{dy}{dt} = ky$. What is the solution for y? $y = Ce^{kt}$, where $C = y(0)$ $y = \frac{k}{2}y^2$ $y = e^{kt} + C$, where $C = y(0)$ $y = C_1 d^{kt} + C_2$, where $C_1 = y(0)$ and C_2 is an arbitrary constant $y = \frac{k}{2}t^2$

Evaluate the following definite integral: $\int_0^3 xe^{x^2} dx$ $\frac{1}{2}(e^9-1)$ $\frac{1}{2}(e^6-1)$ $\frac{1}{2}e^9$ $\frac{1}{2}e^6$ e^6

In order to evaluate the following definite integral

$$\int_4^8 \frac{dx}{x\sqrt{x^2+4}}$$

what substitution should you make as a first step? $x = 2\tan\theta$ $x = 2\sin\theta$ $u = x^2+4$ $u = \sqrt{x^2+4}$ $x = u$