Math 120: Calculus Name:____________________

Exam III Tutorial Instructor:____________________

April 25, 1995 Tutorial Section:____________________

Calculators are *not* allowed on this exam. Hand in this answer page only. Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 19 multiple choice questions, worth 5 points each. An additional 5 points will be given for your correct tutorial section number.

You are taking this exam under the honor code.

What is the sixth degree Taylor polynomial $P_6(x)$ for $f(x) = \sin x$? (Hint: You can use the fact that $P_{2n+1}(x) = \sum_{k=0}^{n} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$ and $P_{2n+2}(x) = P_{2n+1}(x)$, or else just do it from scratch.) $x - \frac{x^3}{3!} + \frac{x^5}{5!}$ $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$ $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!}$ $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \frac{x^{13}}{13!}$ $-x + \frac{x^3}{3!} - \frac{x^5}{5!}$

Find the third degree Taylor polynomial for the function $f(x) = e^{-x}$. $1 - x + \frac{x^2}{2!} - \frac{x^3}{3!}$ $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$ $-1 + x - \frac{x^2}{2!} + \frac{x^3}{3!}$ $-1 - x - \frac{x^2}{2!} - \frac{x^3}{3!}$ $-1 - x - x^2 - x^3$

Find the k^{th}-degree Taylor coefficient of the function $f(x) = \frac{1}{x+1}$ $(-1)^k$ 1 $\frac{1}{k!}$ $\frac{(-1)^k}{k!}$ -1

Which of the following is equal to the binomial coefficient $\binom{8}{3}$? 56 28 112 8 -28

A certain kind of die has four sides, labelled 1 to 4. When the die is rolled, each side has a probability of 1/4 that it will appear on the bottom. If the die is rolled 5 times, what is the probability that the side labelled "3" appears on the bottom in exactly two of the five rolls? $10 \cdot \left(\frac{1}{4}\right)^2 \cdot \left(\frac{3}{4}\right)^3$ $5 \cdot \left(\frac{1}{4}\right)^3 \cdot \left(\frac{3}{4}\right)^2$ $10 \cdot \left(\frac{1}{3}\right)^2 \cdot \left(\frac{2}{3}\right)^3$ $10 \cdot \left(\frac{1}{4}\right)^3 \cdot \left(\frac{3}{4}\right)^2$ $\left(\frac{2}{5}\right)$ $\left(\frac{1}{4}\right)$

Recall Simpson's rule:

$$\int_a^b f(x)\,dx \approx \frac{b-a}{3n}\left[\, f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \ldots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n) \right]$$

Which of the following do you get by applying Simpson's rule with $n = 4$ to the integral

$$\int_1^3 \frac{1}{x}\,dx$$

$\frac{1}{6} \cdot \left[1 + \frac{8}{3} + 1 + \frac{8}{5} + \frac{1}{3}\right]$ $\frac{1}{6} \cdot \left[1 + \frac{2}{3} + \frac{1}{2} + \frac{2}{5} + \frac{1}{3}\right]$ $\frac{1}{6} \cdot \left[1 + \frac{3}{2} + 2 + \frac{5}{2} + 3\right]$ $\frac{1}{6} \cdot \left[1 + \frac{12}{2} + 4 + \frac{20}{2} + 3\right]$ $\frac{1}{2} \cdot \left[1 + \frac{4}{3} + 1 + \frac{4}{5} + \frac{1}{3}\right]$

Recall that associated to the trapezoid rule for approximating $\int_a^b f(x)\,dx$ is the error estimate

$$|\text{error}| \le \frac{M(b-a)^3}{12n^2}$$

where M is the maximum value of $|f''(x)|$ for $a \le x \le b$. Suppose you used the trapezoid rule with $n = 4$ to estimate

$$\int_1^3 \frac{1}{x}\,dx$$

What bound on the error is guaranteed by the error estimate formula? (Note that for this problem you don't need to remember what the trapezoid rule is; you just need the error estimate given above.) $\frac{1}{12}$ $\frac{1}{24}$ $\frac{1}{216}$ $\frac{1}{324}$ 2

Find the length of the curve $y = \frac{2}{3}x^{\frac{3}{2}}$ from $x = 0$ to $x = 3$. $\frac{14}{3}$ $\frac{2}{3}$ $\frac{16}{3}$ $\frac{9}{2}$ $\frac{\sqrt{2}}{3}$

Three objects are placed on the x-axis. The first, having mass 5, is placed at $x = 1$. The second, having mass 8, is placed at $x = 2$. The third, having mass 3, is placed at $x = 3$. Where is the center of mass of the system?

At $x = \frac{15}{8}$ At $x = 2$ At $x = \frac{15}{16}$ At $x = \frac{7}{4}$ At $x = \frac{21}{16}$

Four objects are placed in the (x, y)-plane. They all have the same mass. They are placed at the points $(1, 2)$, $(2, 3)$, $(0, 4)$ and $(1, 7)$. Where is the center of mass of the system? At the point $(1, 4)$ It cannot be computed from the given information At the point $(1, 6)$ At the point $(1, \frac{7}{2})$ At the point $(\frac{3}{2}, \frac{7}{2})$

A lamina (= region of the plane) has the shape of a right isosceles triangle with sides of length 4:

Which of the following represents the x-coordinate of the center of mass (i.e. the centroid) of the lamina? $\frac{1}{8}\int_0^4 (4x - x^2)\,dx$ $\frac{1}{16}\int_0^4 (4x - x^2)\,dx$ $\frac{1}{16}\int_0^4 (4 - x)\,dx$ $\frac{1}{8}\int_0^4 (4 - x)\,dx$ $\frac{1}{8}\int_0^4 (4 - x)^2\,dx$

Set up, but do not evaluate, an integral for the length of the arc of the curve $y^2 = 4\sin x$ from the point $(0,0)$ to the point $(\frac{\pi}{2}, 2)$. (Note that the equation is $y^2 = 4\sin x$, not $y = 4\sin x$.) $\int_0^{\frac{\pi}{2}} \sqrt{1 + \frac{\cos^2 x}{\sin x}}\ dx$ $\int_0^{\frac{\pi}{2}} \sqrt{1 + 16\cos^2 x}\ dx$ $\int_0^{\sqrt{\frac{\pi}{2}}} \sqrt{1 + 16\cos^2 x}\ dx$ $\int_0^{\frac{\pi}{2}} \sqrt{1 + \frac{\cos x}{\sqrt{\sin x}}}\ dx$ $\int_0^{\frac{\pi}{2}} \sqrt{1 + \frac{1}{\sin x}}\ dx$

In the expansion of $(x + y)^{15}$, what is the coefficient of xy^{14}? 15 14 105 1 91

A bloodbank has 10 units of type A$^+$ blood available; but, unknown to them, 5 of the units are contaminated with serum hepatitis. Suppose that 2 units are chosen at random. What is the probability that both units chosen will be contaminated? $\frac{2}{9}$ $\frac{1}{2}$ $\frac{2}{5}$ $\frac{1}{5}$ $\frac{1}{9}$

Set up, but do not evaluate, an integral for the length of the curve $y = x^2$, $3 \le x \le 5$. $\int_3^5 \sqrt{1 + 4x^2}\ dx$ $\int_3^5 \sqrt{1 + 2x}\ dx$ $\int_3^5 \sqrt{1 + x^2}\ dx$ $\int_9^{25} \sqrt{1 + 4x^2}\ dx$ $\int_9^{25} \sqrt{1 + 2x}\ dx$

Recall that for the trapezoid rule to approximate $\int_a^b f(x)\ dx$, the error estimate is

$$|\text{error}| \le \frac{M(b-a)^3}{12n^2}$$

where M is the maximum value of $|f''(x)|$ for $a \le x \le b$. Using this, how large do we have to choose n so that the trapezoid rule approximation to the integral

$$\int_1^3 6 \ln x\ dx$$

is accurate to within 10^{-4}? (Note that for this problem you don't need to remember what the trapezoid rule is; you just need the error estimate given above. The numbers in this problem were chosen so that you do *not* need a calculator.) $n > 200$ $n > 100$ $n > 50$ $n > 10$ $n > 5$

Which of the following is a row of Pascal's triangle? 1 6 15 20 15 6 1 1 6 10 15 10 6 1 1 6 20 35 20 6 1 1 6 15 15 6 1 1 6 10 10 6 1

A certain partial deck of cards contains 6 red cards and 4 black cards. Two cards are chosen at random. What is the probability that both cards are black? $\frac{6}{45}$ $\frac{6}{90}$ $\frac{16}{100}$ $\frac{6}{15}$ $\frac{4}{10}$

Let $f(x) = 3x^5 - x^2 + 1$. Find the degree 4 Taylor polynomial $P_4(x)$ for $f(x)$. $P_4(x) = -x^2 + 1$ $P_4(x) = 3x^5 - x^2 + 1$ $P_4(x) = 1$ $P_4(x) = -x^2$ $P_4(x) = x^4 - x^2 + 1$