Math 120: Calculus Name:________________________

Final Exam Tutorial Instructor:________________________

May 9, 1995 Tutorial Section:________________________

Calculators are *not* allowed on this exam. Hand in this answer page only. Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 29 multiple choice questions, worth 5 points each. An additional 5 points will be given for your correct tutorial section number.

You are taking this exam under the honor code.

Express the repeating decimal 1.259595959... as a fraction. $\frac{1247}{990}$ $\frac{11}{9}$ $\frac{1258}{999}$ $\frac{113}{90}$ $\frac{124}{99}$

Find the sum of the following geometric series:

$$\sum_{k=0}^{\infty} \frac{2^{k+2}}{3^{k+1}}$$

4 1 2 3 5

Joe has six cards: four of them are aces and two are not. If Diane takes away two cards, chosen at random, what is the probability that both are aces? $\frac{2}{5}$ $\frac{3}{5}$ $\frac{2}{3}$ $\frac{1}{3}$ $\frac{1}{2}$

Let $f(x) = \ln(1 - x^2)$. Find the second degree Taylor polynomial $P_2(x)$ of $f(x)$. $-x^2$ $1 - x^2$ $1 + x^2$ x^2 $1 + x + x^2$

Let $f(x) = e^{3x}$. Find the nth Taylor polynomial for $f(x)$. (Hint: You can use the fact that the nth Taylor polynomial for e^x is $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!}$.) $\sum_{k=0}^{n} 3^k \cdot \frac{x^k}{k!}$ $3 \cdot \sum_{k=0}^{n} \frac{x^k}{k!}$ $\sum_{k=0}^{n} \frac{x^k}{(3k)!}$ $\sum_{k=0}^{n} \frac{x^{3k}}{(3k)!}$ $\sum_{k=0}^{n} \frac{x^{3k}}{k!}$

For the following problem you may use any of the following facts (not all will necessarily be used):

$$\int_{-\pi/2}^{\pi/2} \cos^2 x \, dx = \pi/2 \qquad \int_{-\pi/2}^{\pi/2} x \cos x \, dx = 0$$

$$\int_{-\pi/2}^{\pi/2} \cos x \, dx = 2 \qquad \int_{-\pi/2}^{\pi/2} \sin x \cos x \, dx = 0$$

Find the centroid (= center of mass) of the region bounded by the curve $y = \cos x$ and the x-axis, from $x = -\pi/2$ to $x = \pi/2$. $(0, \frac{\pi}{8})$ $(0, \frac{\pi}{4})$ $(0, 0)$ $(0, \frac{\pi}{2})$ $(0, 2)$

Which of the following integrals represents the length of the arc of the curve $y^2 = x^3$ from $(0,0)$ to $(2, \sqrt{8})$? $\int_0^2 \sqrt{1 + \frac{9}{4}x} \, dx$ $\int_0^2 \sqrt{1 + \frac{9}{4x}} \, dx$ $\int_0^2 \sqrt{1 + \frac{3}{2}x^{1/2}} \, dx$ $\int_0^{\sqrt{8}} \sqrt{1 + \frac{9}{4}x} \, dx$ $\int_0^{\sqrt{8}} \sqrt{1 + \frac{3}{2}x^{1/2}} \, dx$

The error bound for Simpson's rule is

$$|\text{error}| \leq \frac{M(b-a)^5}{180n^4}$$

where $M = \max \left| f^{(4)}(x) \right|$, $a \leq x \leq b$. If you apply Simpson's rule with $n = 6$ to the integral

$$\int_1^3 \frac{1}{x}\, dx$$

you get $\frac{2077}{1890}$ as an answer. On the other hand, we know that $\int_1^3 \frac{1}{x}\, dx = \ln 3 - \ln 1$. It follows that

$$\left| \ln 3 - \frac{2077}{1890} \right|$$

is at most which of the following? (Note that the fourth derivative of $\frac{1}{x}$ is $\frac{24}{x^5}$.) $\frac{24 \cdot 2^5}{180 \cdot 6^4}$ $\frac{24 \cdot 2^5}{3^5 \cdot 180 \cdot 6^4}$ $1 + \frac{24 \cdot 2^5}{3^5 \cdot 180 \cdot 6^4}$ $\frac{24 \cdot 3^5}{2^5 \cdot 180 \cdot 6^4}$ $\frac{8 \cdot 2^5}{3^5 \cdot 180 \cdot 6^4}$

Find the following antiderivative:

$$\int \frac{dx}{(x-1)(x+2)}$$

$\frac{1}{3} \ln \left| \frac{x-1}{x+2} \right| + C$ $\frac{1}{3} \ln |(x-1)(x+2)| + C$ $\ln \left| \frac{(x-1)^2}{x+2} \right| + C$ $\frac{1}{3} \left[(x-1)^{-2} + (x+2)^{-2} \right] + C$ $\ln |(x-1)(x+2)| + C$

In order to evaluate the following definite integral:

$$\int_2^{2\sqrt{3}} \frac{dx}{4+x^2}$$

what substitution should you make? $x = 2\tan u$ $u = 2\tan x$ $u = 4 + x^2$ $x = \frac{1}{2}\tan u$ $u = \frac{1}{2}\tan x$

Find the following definite integral:

$$\int_0^{\pi/2} x \sin x\, dx$$

1 0 -1 $-\frac{\pi}{2} + 1$ $-\frac{\pi}{2}$

Find the following antiderivative:

$$\int x^2 \ln x\, dx$$

(Hint: at the end you'll have to pull out a common factor.) $\frac{1}{3}x^3\left[\ln x - \frac{1}{3}\right] + C$ $\frac{1}{3}x^3\left[\ln x + \frac{1}{3}\right] +$
C $x[1 + 2\ln x] + C$ $\frac{1}{3}x^3\left[x\ln x - x\right] + C$ $\frac{1}{3}x^3\left[x\ln x + x\right] + C$

Mary plans to invest some money in a savings account earning 10% interest, compounded continuously. Her goal is to have a total of \$10,000 in the account after 10 years. How much money (in dollars) should she invest now to achieve that goal? $\frac{10,000}{e}$ $\$1,000$ $10,000e$ $\frac{10,000}{e^{10}}$ $10,000e^{10}$

A certain radioactive substance has a half-life of 4 years. If a sample of this substance has an initial mass of 50 kilograms, how much remains after 6 years (in kilograms)? $50 \cdot 2^{-3/2}$ $50 \cdot 4^{-6}$ $50 \cdot 2^{-2/3}$ $50 \cdot 4^{-3/2}$ $\frac{2}{3} \cdot 50$

Find the derivative of $f(x) = 2^x$ $2^x \cdot \ln 2$ 2^x $2 \cdot 2^x$ $e^x \cdot \ln 2$ $x \cdot 2^{x-1}$

Find the domain of the function $f(x) = e^{x^2}\ln(2 - x)$. $-\infty < x < 2$ $-\infty < x \leq 2$ $2 < x < \infty$ $2 \leq x < \infty$ $-\infty < x < 2$ except $x = 0$

Solve for x: $\ln(\ln x) = 1$ $x = e^e$ There is no such x $x = e$ $x = 0$ $x = 1$

Find the derivative of $f(x) = \sqrt{e^{(x^2)}}$. (Hint: $\sqrt{a} = a^{1/2}$ and $(a^b)^c = a^{bc}$) $x\sqrt{e^{(x^2)}}$ e^x $\frac{1}{2\sqrt{e^{(x^2)}}}$ $\frac{x}{\sqrt{e^{(x^2)}}}$ $\frac{2x}{\sqrt{e^{(x^2)}}}$

Evaluate the following integral: $\int_0^1 e^{3x}dx$ $\frac{1}{3}\left(e^3 - 1\right)$ $e^3 - 1$ $\frac{1}{3}e^3$ $\frac{e^3}{3} - 1$ $\frac{1}{3}(e - 1)$

Simplify: $\frac{z^{-1/2}z^2}{z^{1/2}}$ z z^2 $z^{7/2}$ $z^{-3/2}$ $z^{1/2}$

Find the average value of the function $f(x) = x^2$ on the interval $[0, 2]$ $\frac{4}{3}$ $\frac{8}{3}$ 1 2 $\frac{3}{2}$

A tank, shown below, has water in it up to a height of 3 m. Which of the following integrals represents the work required to pump all the water out of the tank? Note that the water is not included in the picture.

$\int_0^3 9800 \cdot 8y(5 - y)\,dy$ $\int_0^3 9800 \cdot \frac{4y}{5}(5 - y)\,dy$ $\int_0^5 9800 \cdot \frac{4y}{5}(3 - y)\,dy$ $\int_0^5 9800 \cdot 8y(3 - y)\,dy$ $\int_0^3 9800 \cdot 4y(5 - y)\,dy$

Which of the following integrals represents the volume of the solid obtained by rotating

about the x-axis the region bounded by the curves $y = e^x$, $x = 2$, $x = 5$ and the x-axis?
$\int_2^5 \pi e^{2x} dx$ $\int_2^5 \pi e^{(x^2)} dx$ $\int_2^5 \pi e^x dx$ $\int_{e^2}^{e^5} \pi e^{2x} dx$ $\int_{e^2}^{e^5} \pi e^{(x^2)} dx$

Which of the following integrals represents the volume of the solid obtained by rotating *about the line* $x = 3$ the region bounded above by $y = x^2$, below by the x-axis, to the left by the y-axis and to the right by the line $x = 3$ (see picture)?

$\int_0^9 \pi(3 - \sqrt{y})^2 dy$ $\int_0^9 \pi \left(\sqrt{y}\right)^2 dy$ $\int_0^3 \pi \left(x^2\right)^2 dx$ $\int_0^3 \pi(3 - x^2)^2 dx$ $\int_0^9 \pi \left(9 - (\sqrt{y})^2\right) dy$

Evaluate the following definite integral: $\int_0^1 2x(x^2 - 1)^8 dx$ $\frac{1}{9}$ $-\frac{1}{9}$ 0 $-\frac{1}{8}$ 1

Let $f(x) = \int_1^{x^2} \sin t\, dt$. What is $f'(x)$? $2x \sin(x^2)$ $2x \cos(x^2)$ $\cos(x^2)$ $\sin(x^2)$ $\sin(2x)$

Over the course of his basketball career, Julius succeeded in making a free throw in 75% (= 3/4) of his attempts. In a randomly selected game, he attempted to make a free throw 12 times. What is the probability that he succeeded in 8 of the 12 attempts? (You don't need to know anything about basketball to answer this problem.)

$\binom{12}{8} \left(\frac{3}{4}\right)^8 \left(\frac{1}{4}\right)^4$ $\frac{2}{3}$ $\frac{\binom{8}{8}}{\binom{12}{8}}$ $\left(\frac{3}{4}\right) \cdot \frac{\binom{8}{8}}{\binom{12}{8}}$ $\frac{3}{4} \cdot \left(\frac{2}{3}\right)^8 \left(\frac{1}{3}\right)^4$

Find the following antiderivative: $\int x \sin(x^2) dx$ $-\frac{1}{2} \cos(x^2) + C$ $-\frac{1}{2} x \cdot \cos(x^2) + C$ $\frac{1}{2} \cos(x^2) + C$ $-2\cos(x^2) + C$ $2\cos(x^2) + C$

A metal chain weighs 2 lbs per ft and is hanging to a length of 20 ft off the top of a tall building. How much work is done in raising the bottom of the chain 15 ft? $\int_0^{15} 2(20 - y) dy$ $\int_5^{20} 2(20 - y) dy$ $\int_0^{20} 2(15 - y) dy$ $\int_0^{15} 2y dy$ $\int_5^{20} 2(15 - y) dy$