

Calculators are not allowed. Hand in this answer page only. Record your answers to the multiple choice problems by placing an \times through one letter for each problem on this answer sheet. There are 19 multiple choice questions, worth 5 points each. An additional 5 points will be given for your correct tutorial section number.

You are taking this exam under the honor code.

Find the slope of the tangent line to the graph of $y = e^{3x}$ at $x = 1$.

$$3e^3 \quad e^3 \quad 1 \quad 3e$$

Simplify the following expression: $\left(\frac{1}{x^{5/2}}\right)^{-3/5}$

$$x^{3/2} \quad \frac{1}{x^{3/2}} \quad x^{15} \quad \frac{1}{x^{2/3}} \quad x^{2/3}$$

$$\text{Find } \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$$

$$\frac{1}{2} \quad \infty \quad 1 \quad 0 \quad 2$$

$$\text{Find } \frac{d}{dx} \ln(e^x + x)$$

$$\frac{e^x + 1}{e^x + x} \quad 1 + \frac{1}{x} \quad \frac{1}{e^x + x} \quad \frac{1}{e^x + 1} \quad \frac{1}{x} \cdot (e^x + x) + \ln(e^x + 1)$$

What are the critical numbers, if any, for the function $f(x) = x + e^x$?

$$\text{There are no critical numbers} \quad x = 0 \quad x = \frac{1}{e} \quad x = e \quad x = 1$$

$$\text{Find } \int_0^{\pi/2} (\cos x)e^{\sin x} dx$$

$$e - 1 \quad e \quad e^{\pi/2} - 1 \quad e^{\pi/2} \quad 0$$

$$\text{Find } \log_{16}(1/4)$$

$$-\frac{1}{2} \quad -2 \quad \frac{1}{2} \quad 2 \quad \frac{1}{4}$$

$$\text{Solve for } x: \ln(2x + 1) = 1$$

$$x = \frac{e - 1}{2} \quad x = 0 \quad x = -\frac{1}{2} \quad x = \frac{1}{2} \quad x = \frac{e}{2} - 1$$

$$\text{Find } \frac{d}{dx} x^{x^2}$$

$$x^{x^2} [x + 2x \ln x] \quad x^{x^2} [2x \ln x] \quad x^{x^2} \quad x^{x^2} [2x] \quad x^{2x}$$

Use logarithmic differentiation to find the derivative $\frac{dy}{dx}$ for $y = (2x + 5)^8(4x - 3)^5$.

$$\frac{dy}{dx} = (2x+5)^8(4x-3)^5 \left[\frac{16}{2x+5} + \frac{20}{4x-3} \right] \frac{dy}{dx} = (2x+5)^8(4x-3)^5 \left[\frac{8}{2x+5} + \frac{5}{4x-3} \right]$$

$$\frac{dy}{dx} = (2x+5)^8(4x-3)^5 \left[\frac{2}{2x+5} + \frac{4}{4x-3} \right] \frac{dy}{dx} = \left[\frac{16}{2x+5} + \frac{20}{4x-3} \right] \frac{dy}{dx} = \left[\frac{8}{2x+5} + \frac{5}{4x-3} \right] \blacksquare$$

You invest a certain amount of money in an account which has an annual interest rate of 10%, compounded continuously. After that you do not make any deposits or withdraw any money from the account. Which of the following is true about the amount of money in the account at the end of 10 years? (**You do not need a calculator for this problem!** Note that “annual interest rate” is what we talked about in class. I do *not* mean “effective annual yield.”)

It is between 2 1/2 and 3 times the original amount. It is between 1 1/2 and 2 times the original amount. It is between 2 and 2 1/2 times the original amount. It is between 3 and 3 1/2 times the original amount. It is between 3 1/2 and 4 times the original amount.

A certain radioactive substance has a half-life of 100 days. You begin with 75 lbs. of the substance. Find the general formula for the amount of the substance that remains after t days.

$$75 \left(\frac{1}{2}\right)^{t/100} \quad 75e^{100t} \quad 75 \left(\frac{1}{2}\right)^{t-99} \quad 75 \left(\frac{1}{2}\right)^t \quad 75 \left(\frac{1}{2}\right)^{100t}$$

$$\text{Find } \lim_{x \rightarrow 0} \frac{x}{e^x} \quad 0 \quad 1 \quad e \quad \frac{1}{e} \quad \infty$$

$$\text{Find } \lim_{x \rightarrow 0} \frac{\ln(1+2x)}{x}$$

$$2 \quad 1 \quad \frac{1}{2} \quad 0 \quad -\infty$$

$$\text{Find } \int_1^e \frac{\ln x}{x} dx$$

$$\frac{1}{2} \quad 1 \quad e - 1 \quad \frac{1}{2}e^2 - \frac{1}{2} \quad \frac{1}{e}$$

Find the equation of the tangent line to the curve $y = x^3e^{-x}$ at the point $(1, \frac{1}{e})$.

$$y - \frac{1}{e} = \frac{2}{e}(x-1) \quad y - \frac{1}{e} = \frac{3}{e}(x-1) \quad y - \frac{1}{e} = \frac{1}{e}(x-1) \quad y - \frac{1}{e} = e(x-1) \quad y - \frac{1}{e} = 2e(x-1)$$

A certain object cools at a rate (in °C/min) equal to 3 times the difference between its temperature and that of the surrounding air. If a room is kept at 20°C and the initial temperature of the object is 50°C, find an expression for the temperature of the object t minutes later. (Hint: if you let $y(t)$ be the temperature of the object t minutes later, then what is $y(0)$?)

$$30e^{3t} + 20 \quad 50e^{3t} + 20 \quad 50e^{3t} \quad 30e^{3t} \quad 50e^{3t} - 20$$

Find the slope of the tangent line to the curve $y = 3^x$ at the point $(2, 9)$.

$$9 \ln 3 \quad \ln 3 \quad \ln 9 \quad \log_3 2 \quad 9$$

On what interval(s) is the function $f(x) = xe^x$ increasing?

$$(-1, \infty) \quad (-\infty, -1) \quad (1, \infty) \quad (-\infty, 1) \quad (-\infty, -1) \quad \text{and} \quad (1, \infty)$$