Math 120: Calculus Name:____________________

Final Exam Tutorial Instructor:____________________

May 8, 1996 Tutorial Section:____________________

Calculators are *not* allowed on this exam. Hand in this answer page only. Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 29 multiple choice questions, worth 5 points each. An additional 5 points will be given for your correct tutorial section number.

You are taking this exam under the honor code.

Find the coefficient of $x^{98}y^2$ in the expansion of $(x+y)^{100}$. 4950 100 5000 98 196

Find $\frac{d}{dx}\int_{3x}^{5} \sin(t^2)\, dt$ $-3\sin(9x^2)$ $3\sin(9x^2)$ $\cos(9x^2)-\cos(25)$ $-3\cos(9x^2)$ $\sin(9x^2)-\sin(25)$

What is the slope of the tangent line to the curve $y = 2^x$ at $x = 0$? $\ln 2$ 1 2 e^2 0

A particle is moving along a line, with acceleration function $a(t) = 6t$ (in m/sec^2) and initial velocity $v(0) = -3$. Find the distance traveled (not the displacement) during the time interval $0 \le t \le 2$. 6 m 2 m 8 m 9 m 12 m

Evaluate $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$ $-2\cos\sqrt{x}+C$ $2\cos\sqrt{x}+C$ $\frac{2}{3}\cos(x^{3/2})+C$ $-\frac{2}{3}\cos(x^{3/2})+C$ $-\cos x + C$

Set up, but do not simplify or evaluate, an integral for the volume of the solid obtained by rotating about the line $y = 3$ the region bounded by $y = e^x$, $y = x$, $x = -1$ and $x = 1$ (shaded in the picture below).

$\int_{-1}^{1} \pi\left[(3-x)^2-(3-e^x)^2\right]\, dx$ $\int_{-1}^{1} \pi\left[3-(e^x-x)^2\right]\, dx$ $\int_{-1}^{1} \pi\left[3-e^{2x}-x^2\right]\, dx$ $\int_{-1}^{1} \pi\left[(3-e^{2x})+\right.$ $\int_{-1}^{1} \pi\left[3-(e^x+x)^2\right]\, dx$

Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating about the y-axis the region between the y-axis and the curve $y = x^2$, from $y = 0$ to $y = 9$ (shaded, below).

$\int_0^9 \pi y\ dy$ $\int_0^9 \frac{1}{2}\pi y^2\ dy$ $\int_0^3 \pi x^4\ dx$ $\int_0^3 \pi\left[81 - x^4\right]\ dx$ $\int_0^9 \pi\left[9 - \sqrt{y}\ \right]\ dy$
Solve for x: $\log_3(\log_2(\log_5 x)) = 1$
5^8 5^9 3^{32} 3^{25} 2^{125}
Let $f(x) = \frac{1}{2}e^{2x} + e^{-x}$. Find the interval(s) on which $f(x)$ is increasing.
$[0,\infty)$ $[-\infty,0]$ $(-\infty,\infty)$ $\left[\frac{1}{3},\infty\right)$ $\left[\frac{1}{\sqrt{2}},\infty\right)$

Find the degree 3 Taylor polynomial for the function $f(x) = \sqrt{1+x}$. (Note that $\sqrt{1+x} = (1+x)^{1/2}$.)

$1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3$ $1 + \frac{1}{2}x - \frac{1}{4}x^2 + \frac{3}{8}x^3$ $1 + \frac{1}{2}x - \frac{1}{2}x^2 + \frac{9}{4}x^3$ $1 + \frac{1}{2}x + \frac{1}{4}x^2 + \frac{1}{8}x^3$ $1 - \frac{1}{2}x + \frac{1}{4}x^2 - \frac{1}{8}x^3$

Find the nth Taylor polynomial for $g(x) = e^{-2x}$. (You can either compute it directly, or else use the fact that the nth Taylor polynomial for $f(x) = e^x$ is $1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} = \sum_{k=0}^{n} \frac{x^k}{k!}$.)

$\sum_{k=0}^{n}(-1)^k 2^k \frac{x^k}{k!}$ $\sum_{k=0}^{n} 2^k \frac{x^k}{k!}$ $\sum_{k=0}^{n}(-1)^k \frac{x^k}{k!}$ $\sum_{k=0}^{n} 2^k \frac{x^{-k}}{k!}$ $\sum_{k=0}^{n} \frac{x^{-2k}}{k!}$

Giuseppe's Pizza has 8 different toppings that they can put on their pizzas. A customer ordering their Deluxe pizza can choose any 3 of the toppings. How many different Deluxe pizzas can be made? (For example, a pizza consisting of pepperoni, sausage and green pepper is different from one consisting of pepperoni, sausage and mushrooms.)

56 28 24 3 48

A fair die has 6 sides, of which two are painted white and four are painted black. Jack is told to roll the die a total of five times, each time recording whether it comes up white or black. He is told that if he rolls at least four whites, he will win \$1,000,000. What is the probability that he will win \$1,000,000?

$\frac{11}{243}$ $\frac{10}{243}$ $\frac{3}{243}$ $\frac{112}{243}$ $\frac{6}{243}$
Find $\lim_{x\to-\infty} x^2 e^x$.
0 ∞ $-\infty$ 2 -2
Find $\lim_{x\to 1} \frac{x^8 - 1}{x^5 - 1}$, if it exists.
$\frac{8}{5}$ 1 0 ∞ limit does not exist
Evaluate $\int \frac{x^4 + 1}{x^2 - 1}\ dx$.

$\frac{1}{3}x^3 + x + \ln(x-1) - \ln(x+1) + C$ $\frac{1}{3}x^3 - x + \ln(x-1) + \ln(x+1) + C$ $\frac{1}{3}x^3 - x + C$ $\frac{1}{3}x^3 + x + C$ $\frac{1}{3}x^3 + x + 2\ln(x-1) + 2\ln(x+1) + C$

It is a fact, which you don't have to verify, that

$$\frac{4x^2}{x^4-2x^2+1} = \frac{1}{x-1} + \frac{1}{(x-1)^2} - \frac{1}{x+1} + \frac{1}{(x+1)^2}$$

(Note that the third term is a minus.) Using this fact, evaluate $\displaystyle\int \frac{4x^2}{x^4-2x^2+1}\,dx$.

$\ln\left|\frac{x-1}{x+1}\right| - \frac{1}{x-1} - \frac{1}{x+1} + C$ $\ln\left|\frac{x-1}{x+1}\right| + \frac{1}{x-1} + \frac{1}{x+1} + C$ $\ln|(x-1)(x+1)| + \frac{1}{x-1} + \frac{1}{x+1} + C$ $\ln|(x-1)(x+1)| - \frac{2}{(x-1)^3} - \frac{2}{(x+1)^3} + C$ $\ln\left|\frac{x-1}{x+1}\right| + \frac{2}{(x-1)^3} + \frac{2}{(x+1)^3} + C$

Evaluate $\displaystyle\int_0^{\pi/2} x\cos(2x)\,dx$

$-\frac{1}{2}$ 0 $\frac{1}{2}$ 1 $\frac{3}{2}$

Find the average value of the function $f(x) = x^2$ on the interval $[0, 2]$.

$\frac{4}{3}$ $\frac{8}{3}$ 2 1 $\frac{3}{2}$

If you were to use a trigonometric substitution to evaluate the integral $\displaystyle\int \frac{x^2}{\sqrt{5+x^2}}\,dx$, what substitution should you use?

$x = \sqrt{5}\tan\theta$ $x = 5\tan\theta$ $x = \sqrt{5}\sin\theta$ $x = 5\sin\theta$ $x = 5\sec\theta$

Find the equation of the tangent line to the curve $y = \ln(x^2+1)$ at the point $(1, \ln 2)$.

$y = x - 1 + \ln 2$ $y = 2x + \ln 2$ $y = x - 1$ $y = 2x - 2 + \ln 2$ $y = x - \ln 2$

A certain sum of money is deposited in an account which earns an annual interest rate of 10%, compounded continuously. No further deposit is made into the account. How long does it take (in years) before the total amount of money in the account is twice the original amount?

$10\cdot\ln 2$ (approx. 6.9) 10 5 $2\cdot\ln 10$ (approx. 4.6) It depends on how much was originally deposited.

The initial amount of a certain radioactive substance is 100g. After 3 minutes, 50g remains. How much remains after 4 minutes?

$100\left(\frac{1}{2}\right)^{4/3}$ $100e^{4/3}$ $100\left(\frac{1}{e}\right)^{4/3}$ $100\left(\frac{1}{2}\right)^{4}$ $100\left(\frac{1}{3}\right)^{4}$

Evaluate $\displaystyle\int_{\pi/2}^{\pi} \frac{\sin x}{2+\cos x}\,dx$.

$\ln 2$ 1 0 $\ln 3$ $\ln 3 - \ln 2$

Suppose that a certain spring requires 2 J of work to stretch from its natural length of 0.30 m to a length of 0.40 m. What is the constant in Hooke's Law for this spring? (I.e. Hooke's Law says that the force is proportional to the displacement, $f(x) = kx$. I'm asking what k is in this case.)

400 $\frac{400}{7}$ 25 4 $\frac{4}{3}$

A tank has the shape of an inverted circular cone with height 10 m and radius 5 m. It is filled with water to a height of 7 m. Which of the following integrals represents the work required to empty the tank by pumping all the water to the top of the tank. (The density of water is 1000 kg/m^3 and the acceleration due to gravity is 9.8 m/sec^2. Notice that the answers don't all have the same limits of integration, but they all assume that x is measured from the *top* of the tank, as indicated in the diagram.)

$\int_3^{10} 9800\pi \left(\frac{10-x}{2}\right)^2 x\, dx$ $\int_3^{10} 9800\pi \left(\frac{x}{2}\right)^2 (10-x)\, dx$ $\int_0^{7} 9800\pi \left(\frac{x}{2}\right)^2 x\, dx$ $\int_0^{3} 9800\pi \left(\frac{10-x}{2}\right)^2 x$
$\int_0^{3} 9800\pi \left(\frac{10-x}{2}\right)^2 (10-x)\, dx$

Find $\lim_{x\to 1^-} e^{2/(x-1)}$

0 ∞ $-\infty$ 1 e

Find $\int \cos^3 4x\, dx$

$\frac{1}{4}\sin 4x - \frac{1}{12}\sin^3 4x + C$ $\frac{1}{16}\cos^4 4x + C$ $-\frac{1}{4}\sin 4x + \frac{1}{12}\sin^3 4x + C$ $\sin 4x - \frac{1}{3}\sin^3 4x + C$
$\frac{1}{4}\cos^4 4x + C$

Find the slope of the tangent line to the curve $y = e^{-x^2}$ at the point $(1, \frac{1}{e})$.

$-\frac{2}{e}$ $\frac{1}{e}$ $\frac{2}{e}$ 0 $-\frac{1}{e}$

I enjoyed teaching this course. Have a great summer vacation, and good luck next year!!