Math 120: Calculus Name:__________

Exam II MWF Instructor:__________

March 6, 1997 Tutorial Section:__________

Calculators are not allowed. Hand in this answer page only. Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 20 multiple choice questions, worth 5 points each.

You are taking this exam under the honor code.

Find $\lim_{x\to 0} \dfrac{2^x + 2x - 1}{x}$ $\ln(2)+2$ -1 3 $\dfrac{1}{\ln(2)}+2$ $\log_2(0)+2$

Find the slope of the tangent line to the graph of $y = x^x$ at $x = e$.
$2e^e$ $e\cdot e^{e-1}$ $e^e(e+1)$ e^e $e\cdot e^e$

Differentiate $g(t) = \sqrt{e^{t^2}+1}$
$\dfrac{te^{t^2}}{\sqrt{e^{t^2}+1}}$ $\dfrac{t}{\sqrt{e^{t^2}+1}}$ $\dfrac{e^{t^2}}{\sqrt{e^{t^2}+1}}$ $\dfrac{e^{t^2}}{2\sqrt{e^{t^2}+1}}$ $\dfrac{1}{2\sqrt{e^{t^2}+1}}$

Differentiate $f(x) = \dfrac{e^x+1}{e^x-1}$
$\dfrac{-2e^x}{(e^x-1)^2}$ 0 $\dfrac{2e^x}{(e^x-1)^2}$ $\dfrac{2}{(e^x-1)^2}$ $\dfrac{2e^{2x}}{(e^x-1)^2}$

On what interval(s) is the function $\dfrac{e^x}{x}$ decreasing?
only $(-\infty,0)$ and $(0,1)$ only $(1,\infty)$ only $(0,1)$ only $(-\infty,0)$ it is never decreasing

Solve for x: $2\log_5 x - \log_5 3 = 1$
$x=\sqrt{15}$ $x = 5^{3/2}$ $x = \dfrac{15}{2}$ $x = 30$ $x = 4$

Simplify $e^{t^2(\ln 3)/4}$
$3^{t^2/4}$ $\left(\dfrac{t^2}{4}\right)^3$ $\left(\dfrac{3t^2}{4}\right)$ $3\ln\left(\dfrac{t^2}{4}\right)$ $3e^{t^2/4}$

Suppose that A, B and C are numbers satisfying

$$\log_{10}\left(\frac{A}{C}\right) = 3.5 \qquad \text{and} \qquad \log_{10}\left(\frac{B}{C}\right) = 2.1$$

Find $\dfrac{A}{B}$.
$10^{1.4}$ $\dfrac{3.5}{2.1}$ $10^{3.5/2.1}$ 1.4 $10^{3.5} - 10^{2.1}$

Find $\dfrac{d}{dx}\ln\left(\sqrt{x^2+1}\right)$
$\dfrac{x}{x^2+1}$ $\dfrac{2x}{x^2+1}$ $\dfrac{1}{\sqrt{x^2+1}}$ $\dfrac{x}{2(x^2+1)}$ $\dfrac{2x}{\sqrt{x^2+1}}$

A bacteria culture grows at a rate proportional to its size. Suppose that the initial size is 8000 bacteria, and after 2 hours there are 12,000 bacteria. How many bacteria are there after 6 hours?

27,000 20,000 24,000 36,000 48,000

A certain radioactive substance is known to have a half-life of 10 years. If you have a certain initial amount of this substance, how many years will elapse before only 1/3 of it remains? (The following answers are all in years.)

$\dfrac{10\ln 3}{\ln 2}$ $\dfrac{10\ln 2}{\ln 3}$ $10(\ln 3 - \ln 2)$ $\dfrac{20}{3}$ It is impossible to tell without knowing the initial amount.

Fred plans to invest money into an account which pays an annual interest rate of 10%, compounded continuously. How much should he invest if he does not plan to make any further investments into this account, and he wants the balance after two years to be $20,000?

$\dfrac{20,000}{e^{0.2}}$ $20,000e^{0.2}$ 16,000 16,200 $\dfrac{40,000}{e^{0.1}}$

Suppose that $y(t)$ is a solution to the differential equation $\dfrac{dy}{dt} = 5y$, and suppose that $y(1) = 100$. (Note that this is $y(1)$, not $y(0)$.) Find $y(t)$.

$100e^{5(t-1)}$ $5e^{100t}$ $5e^{100}$ $\dfrac{e^{5t}}{100}$ $100e^{5}$

Evaluate $\displaystyle\int_1^4 \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 dx$

$\dfrac{27}{2} + \ln 4$ $\dfrac{33}{2} + \ln 4$ $\dfrac{37}{2} + \ln 4$ $\dfrac{41}{2} + \ln 4$ $\dfrac{51}{2} + \ln 4$

Evaluate $\displaystyle\int_0^1 2^x \, dx$

$\dfrac{1}{\ln 2}$ $\dfrac{2}{\ln 2}$ $\ln 2$ $2\ln 2$ $\dfrac{3}{\ln 2}$

Use logarithmic differentiation to find $\dfrac{dy}{dx}$ for $y = \dfrac{x^5\sqrt{x^2+1}}{\sin^4 x}$

$\dfrac{dy}{dx} = \dfrac{x^5\sqrt{x^2+1}}{\sin^4 x}\left[\dfrac{5}{x} + \dfrac{x}{x^2+1} - \dfrac{4\cos x}{\sin x}\right]$ $\dfrac{dy}{dx} = \dfrac{x^5\sqrt{x^2+1}}{\sin^4 x}\left[\dfrac{5}{x} + \dfrac{1}{2(x^2+1)} - \dfrac{4\cos x}{\sin x}\right]$
$\dfrac{dy}{dx} = \dfrac{x^5\sqrt{x^2+1}}{\sin^4 x}\left[5x^4 + x(x^2+1)^{-1/2} - 4\sin^3 x\cos x\right]$ $\dfrac{dy}{dx} = \dfrac{x^5\sqrt{x^2+1}}{\sin^4 x}\left[\dfrac{(5x^4)[x(x^2+1)^{-1/2}]}{4\sin^3 x\cos x}\right]$
$\dfrac{dy}{dx} = \dfrac{x^5\sqrt{x^2+1}}{\sin^4 x}\left[\dfrac{5}{x} + \dfrac{2}{x^2+1} + \dfrac{4\cos x}{\sin x}\right]$

Find $\displaystyle\lim_{x\to 0^+} x\ln x$.

0 1 e ∞ $-\infty$

Evaluate $\displaystyle\int \frac{x}{e^{x^2}}dx$.

$\dfrac{-1}{2e^{x^2}} + C$ $\dfrac{1}{2e^{x^2}} + C$ $\dfrac{-1}{e^{x^2}} + C$ $\dfrac{1}{e^{x^2}} + C$ $\dfrac{x^2}{2e^{x^3}} + C$

Simplify: $\log_{\frac{1}{2}} 16$

-4 4 $\dfrac{1}{4}$ $\dfrac{-1}{4}$ 8

Find the slope of the tangent line to the curve $y = \log_5 x$ at $x = 2$.

$\frac{1}{2\ln 5}$ $2\ln 5$ $\frac{2}{\ln 5}$ $\frac{\ln 5}{2}$ $\ln 5$