Math 120: Calculus Name:_____

Final Exam MWF Instructor:_____

May 8, 1997 Tutorial Section:_____

Calculators are not allowed. Hand in this answer page only. Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 25 multiple choice questions, worth 6 points each.

You are taking this exam under the honor code.

What is the coefficient of x^8y^2 in the expansion of $(x+y)^{10}$?

45 10 8 90 10!

Suppose that 1/3 of the people in a certain population were born in a month that ends in the letter "r." Suppose that three people are chosen at random. What is the probability that exactly one of the three was born in a month ending in the letter "r?"

4/9 1/3 1 2/3 1/2

Find the slope of the tangent line to the graph of $y = e^{\sin x}$ at $x = \pi/2$.

0 1 e -1 $1/e$

On what interval(s) is the function $f(x) = xe^{x^2}$ increasing?

$(-\infty, \infty)$ $(0, \infty)$ $(-\infty, 0)$ It is never increasing $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

Simplify $\log_2 48 - \log_2 3$

4 $\log_2 45$ 16 2 8

If $f(x) = \dfrac{\ln x}{x}$, find $f'(e)$.

0 $\dfrac{1}{e}$ $\dfrac{1}{e^2}$ $-\dfrac{1}{e}$ $-\dfrac{1}{e^2}$

Use logarithmic differentiation to find $\dfrac{dy}{dx}$ for $y = \dfrac{e^{x^2}\sqrt{x^2+1}}{x^5}$

$\dfrac{dy}{dx} = \dfrac{e^{x^2}\sqrt{x^2+1}}{x^5}\left[2x + \dfrac{x}{x^2+1} - \dfrac{5}{x}\right]$ $\dfrac{dy}{dx} = \dfrac{e^{x^2}\sqrt{x^2+1}}{x^5}\left[\dfrac{2x}{e^{x^2}} + \dfrac{x}{x^2+1} - \dfrac{5}{x}\right]$ $\dfrac{dy}{dx} = \dfrac{e^{x^2}\sqrt{x^2+1}}{x^5}\left[\dfrac{2x}{e^{x^2}} + \dfrac{2x}{x^2+1} - 5x^4\right]$ $\dfrac{dy}{dx} = \dfrac{e^{x^2}\sqrt{x^2+1}}{x^5}\left[2x + \dfrac{2x}{x^2+1} - \dfrac{5}{x}\right]$ $\dfrac{dy}{dx} = \dfrac{e^{x^2}\sqrt{x^2+1}}{x^5}\left[2x + \dfrac{2x}{x^2+1}\right.$

Find $\lim\limits_{x\to 0} \dfrac{\cos x - 1}{e^x - 1}$

0 1 -1 The limit does not exist. $-\dfrac{1}{2}$

Fred invests \$1000 in an account which offers continuous compounding. After exactly 8 years, the amount of money in the account has tripled to \$3000. What annual interest rate is he getting in his account?

$\dfrac{1}{8}\ln 3$ $8\ln 3$ $\dfrac{1}{3}\ln 8$ $3\ln 8$ $24\ln 2$

A certain substance has a half-life of 4 hours. If a sample has an initial mass of 20 mg, after how many hours will only 15 mg remain?

$\dfrac{4\ln(3/4)}{\ln(1/2)}$ $\dfrac{3\ln(1/4)}{\ln(1/2)}$ $\dfrac{4\ln(1/4)}{\ln(1/2)}$ $\dfrac{4\ln(1/3)}{\ln(1/2)}$ $\dfrac{\ln(4/3)}{\ln(1/2)}$

Recall that $\int \ln x \; dx = x \ln x - x + C$. Using this fact, find $\int_1^5 \log_5 x \; dx$.

$5 - \dfrac{4}{\ln 5}$ $1 \; \ln 5 - 1$ $4 \ln 5 - 1$ $5 \ln 5 - 4$

Let R be the region bounded by $y = \sin x$ and the x-axis, from $x = 0$ to $x = \pi$. Find the volume of the solid generated by rotating R about the y-axis.

$2\pi^2$ π^2 $\pi + 2$ $2\pi^2 + \pi$ $\pi^2 - \pi$

Evaluate: $\int_0^{\pi/2} \sin^3 x \; dx$.

$\dfrac{2}{3}$ π 1 $\dfrac{1}{3}$ $\pi - 1$

Evaluate: $\int_2^3 \dfrac{2x^3}{x^2 - 1} dx$

$5 + \ln\left(\dfrac{8}{3}\right)$ $5 + \ln(3)$ $5 + \ln\left(\dfrac{9}{4}\right)$ $3 + \ln\left(\dfrac{2}{3}\right)$ $3 + \ln\left(\dfrac{7}{2}\right)$

Find the length of the curve $y = \int_0^x \sqrt{t^2 + 4t + 3} \; dt$ from $x = 1$ to $x = 3$.

8 2 10 6 12

Solve: $\int x^3 e^{x^2} \; dx$

$\dfrac{1}{2}\left(x^2 e^{x^2} - e^{x^2}\right) + C$

$\dfrac{1}{3}\left(x^3 e^{x^2} + 3e^{x^2}\right) + C$

$\dfrac{1}{6}\left(x^3 e^{x^2} - 3x^2 e^{x^2} + 6x e^{x^2} - 6e^{x^2}\right) + C$

$\dfrac{1}{2}\left(x^3 e^{x^2} - 6e^{x^2}\right) + C$

$\dfrac{1}{6}\left(x^3 e^{x^2} - 6x e^{x^2}\right) + C$

Let $f(x) = \sqrt{x + 4}$. Find the second degree Taylor polynomial for $f(x)$ and evaluate it at $x = 1$ (i.e plug $x = 1$ into the second degree Taylor polynomial).

$\dfrac{143}{64}$ $\dfrac{17}{8}$ $\sqrt{5} + \dfrac{7}{8}$ $\dfrac{23}{8}$ $\dfrac{45}{16}$

Find the sum of the infinite series: $\dfrac{4}{3} - \dfrac{4}{9} + \dfrac{4}{27} - \dfrac{4}{81} + \dfrac{4}{243} - \dfrac{4}{729} + \cdots$

1 $\dfrac{5}{3}$ $\dfrac{6}{5}$ $\dfrac{5}{4}$ $\dfrac{2}{3}$

Find the area bounded by the graphs of the following equations

$$y = \frac{1}{x^2}, \; y = -x^2, \; x = 1, \; x = 2$$

17/6

13/5

19/4

11/3

21/7

Let R be the region bounded by the graphs of $y = x^3$, and $y = x^2$. Find the volume of the solid generated by rotating R about the x-axis.

$(2/35)\pi$
$(4/37)\pi$
$(39/121)\pi$
$(141/2)\pi$
$(3/2)\pi$

A spring of natural length 10 in. stretches 1.5 in. under a force of 8 lb. Find the work done in stretching the spring from a length of 11 in. to a length of 13 in.

64/3 in.-lb
59/3 in.-lb
58/3 in.-lb
61/3 in.-lb
63/3 in.-lb

A fishtank has a rectangular base of width 2 ft and length 4 ft. The sides are rectangular and have height 3 ft. If the tank is filled with water weighing 62.5 lb/ft^3, find the work required to pump all the water over the top of the tank.

2250 ft-lb
1145 ft-lb
1235 ft-lb
3321 ft-lb
2567 ft-lb

Let $f(x) = 3\sqrt{x+1}$. Find the average value of $f(x)$ on the interval $[-1, 8]$.

6
5.5
6.5
4
4.5

Let

$$f(x) = \int_2^{x^4} \frac{t}{\sqrt{t^3+2}} dt$$

Find $f'(x)$.

$4x^7/\sqrt{x^{12}+2}$
$x/\sqrt{x^4+2}$
$4x^7/\sqrt{x^7+2}$
$x^4/\sqrt{x^4+2}$
$x^4/\sqrt{x^{12}+2}$

Let R be the region bounded by the graphs of $y = 1/x, x = 1, x = 3$ and $y = 0$. Find the volume of the solid generated by rotating R about the x-axis.

$2\pi/3$
$3\pi/2$
2π
3π
$2\pi/5$