[12pt]article document
Your name:

Final Exam., Math. 120, Fall, 1999

Instructions: The exam. consists of 22 problems. There is space to work beside each problem. Most of the problems offer multiple-choice answers. You should use the answers only to check yourself, not as a means of finding an answer. It is important to show your work, and make your steps clear and readable. Partial credit may be given for solutions that are set up correctly, even though the final answer is incorrect, and full credit may not be given for a correct answer that is not supported by correct work.

Please remember your obligations under the honor code.

The spaces below are for recording scores, not for answers.

1. 2.5 in 12.

2. 2.5 in 13.

3. 2.5 in 14.

4. 2.5 in 15.

5. 2.5 in 16.

6. 2.5 in 17.

7. 2.5 in 18.

8. 2.5 in 19.

9. 2.5 in 20.

10.2.5 in 21.

11.2.5 in 22.
1. If f is continuous on the interval $[1, 3]$ and $2 \leq f(x) \leq 4$, what can you say about $\int_1^3 f(x)\,dx$?
(a) $1 \leq \int_1^3 f(x)\,dx \leq 5$ .5in (b) $2 \leq \int_1^3 f(x)\,dx \leq 6$ .5in
(c) $3 \leq \int_1^3 f(x)\,dx \leq 7$ .5in (d) $4 \leq \int_1^3 f(x)\,dx \leq 8$
(e) it is impossible to say anything without knowing the function
3in
2. If $F(x) = \int_0^{\sin x} e^{t^2}\,dt$, what is $F'(x)$?
(a) e^{x^2}.3in (b) $e^{\sin^2 x} \cos x$.3in (c) $e^x \cos x$.3in (d) $e^{x^2} \cos x$.3in (e) $e^{\cos x} \sin x$
3. Find $\int_0^{\pi 2} \sin^2 x \cos x\,dx$.
(a) 0 .5in (b) 112 .5in (c) 16 .5in (d) 14 .5in (e) 13
3in
4. What is the area of the region between the curves $y = e^x$ and $y = x$, for $0 \leq x \leq 1$?
(a) $e - 32$ .3in (b) $2e3$ .3in (c) $1 - 1e$ .3in (d) $e + 12$ .3in (e) $1 + e2$
5. The region bounded by $y = e^x$, $y = x$, $x = 0$, and $x = 1$ is rotated about the x-axis. What is the volume of the resulting solid of revolution?
(a) $2\pi e^2$.5in (b) $12(3e^2 + 5)$.5in (c) $\pi 6(3e^2 - 5)$.5in
(d) $\pi 3(e^2 + 1)$.8in (e) $32(e + 1)$
3in

1

6. If it requires a force of 2 lbs. to hold a spring at 1 ft. beyond its natural length, what is the work (in ft.-lbs.) done in stretching the spring from its natural length to 3 ft. beyond that length?

(a) 5 (b) 6 (c) 7 (d) 8 (e) 9

7. Complete the statement of the theorem below.

Mean-value Theorem for Integrals: Let f be continuous on the closed interval $[a, b]$. Then there exists c in the open interval (a, b) such that

$$\int_a^b f(x)\, dx =$$

8. What is $\frac{d}{dx} x^{2x}$?

(a) x^{2x-1} (b) x^{2x} (c) $x^{2x}(2 + 2\ln x)$

(d) $2x(1 - \ln x)$ (e) $2x^{2x-1}$

9. What is $\int_1^e \frac{1}{x}\, dx$?

(a) 0 (b) 1 (c) e (d) $1 - e$ (e) $e - 1$

10. If $y = e^x(x + 1)^6(x - 3)^2$, what is y'?

(a) $e^x(x + 1)^6(x - 3)^2(1 + 6x + 1 - 2x - 3)$ (b) $(1 + 6x + 1 - 2x - 3)$ (c) $3e^x(x + 1)^5 x - 3$

(d) $e^x + 6(x + 1)^5 - 2(x - 3)$ (e) $e^x + 6\ln(x + 1) - 2\ln(x - 3)$

11. A radioactive substance has a half-life of 10 years. How long (in years) will it take for a sample of 10 g. to decay to 1 g.?

(a) $\ln 2 10$ (b) $10\ln 10 \ln 2$ (c) $2\ln 10$ (d) $10\ln 2$ (e) $2\ln 10$

12. What is $\lim_{x \to 0} \frac{\sin 2x}{x}$?

(a) 0 (b) $\frac{\pi}{2}$ (c) 1 (d) 2π (e) 2

13. Find $\int_0^1 xe^x\, dx$.

(a) 1 (b) 2 (c) e (d) $2e$ (e) $2e - 1$

14. Find $\int_4^5 x(x - 2)(x - 3)\, dx$.

(a) $5\ln 2 - 2\ln 3$ (b) $7\ln 3 + 4\ln 2$ (c) $3\ln 2 + 2\ln 3$

(d) $5\ln 3 - 2\ln 2$ (e) $7\ln 2 - 2\ln 3$

15. What is $\int_0^\pi \sin^2 x\, dx$?

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{8}$ (c) $\frac{\pi}{12}$ (d) $\frac{\pi}{24}$ (e) 2π

16. What is the centroid of the region bounded by $y = x^2$ and $y = x^3$?

(a) $(45, 17)$ (b) $(35, 1235)$ (c) $(54, 73)$ (d) $(120, 135)$ (e) $(53, 59)$

17. Which of the following integrals represents the length of the curve $y = x^2$, for $0 \le x \le 12$?

(a) $\int_0^{12}(1 + 4x^2)\, dx$ (b) $\int_0^{12}\sqrt{1 + 2x}\, dx$ (c) $\int_0^{12}\sqrt{1 + 4x^2}\, dx$

(d) $\int_0^{12}(1 + 2x)\, dx$ (e) $\int_0^{12}\sqrt{1 + x^4}\, dx$

18. An appropriate substitution transforms the integral $\int_0^{12}\sqrt{1 + 4x^2}\, dx$ into which of the following?

(a) $\int_0^{\pi 4} 12\sec^3\theta\, d\theta$ (b) $\int_0^{12}\sec\theta\, d\theta$ (c) $\int_0^{12}\sin^3\theta\, d\theta$

(d) $\int_{\pi 6}^{\pi 2}\sec^3\theta\, d\theta$ (e) $\int_{\pi 6}^{\pi 2} 12\sec\theta\, d\theta$

19. If a fair die is rolled four times, what is the probability that it will come up 1 exactly twice?

(a) 23216 (b) 19 (c) 25216 (d) 13108 (e) 18

20. What is $1 + 2 + 2^2 + \ldots + 2^{56}$?

(a) $2^{57} - 1$ (b) $2^{56} + 1$ (c) $2^{55} - 1$ (d) $2^{56} + 1$

(e) there is no sum, since $|2| \ge 1$

21. For the function $f(x) = \cos x$, find the 5th Taylor polynomial $P_5(x)$.

(a) $1 + x + x^2 + x^3 + x^4 + x^5$ (b) $1 + x + 12x^2 + 16x^3 + 124x^4 + 1120x^5$

(c) $1 - 12x^2 + 124x^4$ (d) $x - 16x^3 + 1120x^5$ (e) $1 + x + 12x^2 + 13x^3 + 14x^4 + 15x^5$

22. Find the Taylor series for $f(x) = e^{x^2}$.
(a) $\sum_{k=0}^{\infty} x^{2k} k!.5$in (b) $\sum_{k=0}^{\infty} x^k k!.5$in (c) $\sum_{k=0}^{\infty} x^{2k} (2k)!.5$in (d) $\sum_{k=0}^{\infty} x^{2k+1} (2k+1)!$
(e) the function has no Taylor series