Math 120 – Final Exam
Friday, December 15, 2000

Name:______________________________ **Section:**____________

This examination consists of 30 problems, worth a total of 150 points on 11 pages, including this front cover. If problems are missing from your copy, you must ask for a new copy right away. All of the problems are multiple choice **with no partial credit for any reason**. Be sure to indicate your single answer to each question by placing an × through that letter on the answer grid below. Students will **NOT** be allowed extra time to fill in the grid after the exam has ended if they forget to do so during the exam!

1.	a	b	c	d	e	**11.**	a	b	c	d	e	**21.**	a	b	c	d	e
2.	a	b	c	d	e	**12.**	a	b	c	d	e	**22.**	a	b	c	d	e
3.	a	b	c	d	e	**13.**	a	b	c	d	e	**23.**	a	b	c	d	e
4.	a	b	c	d	e	**14.**	a	b	c	d	e	**24.**	a	b	c	d	e
5.	a	b	c	d	e	**15.**	a	b	c	d	e	**25.**	a	b	c	d	e
6.	a	b	c	d	e	**16.**	a	b	c	d	e	**26.**	a	b	c	d	e
7.	a	b	c	d	e	**17.**	a	b	c	d	e	**27.**	a	b	c	d	e
8.	a	b	c	d	e	**18.**	a	b	c	d	e	**28.**	a	b	c	d	e
9.	a	b	c	d	e	**19.**	a	b	c	d	e	**29.**	a	b	c	d	e
10.	a	b	c	d	e	**20.**	a	b	c	d	e	**30.**	a	b	c	d	e

Sign the following honor code statement:
"On my honor, I have neither given nor received unauthorized aid on this exam." ______________________________

1. Suppose $x^3 \leq f(x) \leq x^2$ for $-1 \leq x \leq 1$. Which of the following must be true

(a) $f(x)$ is always positive.

(b) $0 \leq \int_{-1}^{1} f(x)\, dx \leq \frac{2}{3}$

(c) $\frac{1}{4} \leq \int_{-1}^{1} f(x)\, dx \leq \frac{1}{3}$

(d) $\int_{-1}^{1} f(x)\, dx$ is negative.

(e) $f(1) = 0$

2. Find the area of the region enclosed by $y = x$ and $y = \sqrt{x}$.

(a) $\frac{1}{2}$ (b) -1 (c) 0 (d) $\frac{1}{6}$ (e) 4

3. Suppose the natural length of a spring is 40cm and a 20N force is required to hold it at 50cm. Find the work required to stretch the spring from 40cm to 50cm.

(a) 1J (b) .001J (c) 1000J (d) 50J (e) .05J

4. Compute the average value of $sin(x)$ over the interval from $x = 0$ to $x = \pi$.

(a) 0 (b) $\frac{\pi}{2}$ (c) 1 (d) $\frac{2}{\pi}$ (e) 2π

5. Which of the following is equal to the derivative of $f(x) = ln|sin(x)|$?

(a) $sin^2(x)$ (b) $tan(x)$ (c) $sec(x)$

(d) $csc(x)$ (e) $cot(x)$

6. Compute

$$\int_1^2 \frac{e^{\frac{1}{x}}}{x^2}\, dx.$$

(a) 1 (b) $\frac{e}{2} - e^{\frac{1}{2}}$ (c) $e - \sqrt{e}$

(d) $e^{\frac{1}{x}}$ (e) $\frac{1}{4}$

7. Which of the following is a Riemann Sum approximating the area under the curve $y = \sqrt{x+3}$ between $x = 1$ and $x = 3$?

(a) $\int_1^3 \sqrt{x+3}\ dx$ (b) $\sum_{k=1}^4 \frac{1}{2}\sqrt{\frac{k}{2}}$ (c) $\sum_{k=1}^4 \frac{1}{2}\sqrt{4+\frac{k}{2}}$

(d) $1+\sqrt{2}$ (e) $\frac{1}{2}(1+\sqrt{2}+\sqrt{3}+\sqrt{4})$

8. According to the Fundamental Theorem of Calculus, what is $g'(x)$ when $g(x) = \int_{x^2}^2 t - 3\ dt$.

(a) $6x - 2x^3$ (b) $\frac{(x^2-3)^2}{2}$ (c) $3x^2 - 6x$

(d) $4t^3 - 6$ (e) $2t$

9. Compute the volume of the solid generated by rotating the region between $y = x^2$ and $y = x^3$ around the y-axis.

(a) π (b) $\frac{\pi}{10}$ (c) $\frac{1}{10}$ (d) $\frac{1}{20}$ (e) 2π

10. Which of the following represents the work done by splashing all of the water out of a rectangular tub of length 5m, width 2m and depth 3m?

(a) $\int_0^2 9800x^4\ dx$ (b) $\int_2^5 98000x\ dx$ (c) $\int_3^5 98000x^4\ dx$

(d) $\int_2^3 98000x^5\ dx$ (e) $\int_0^3 98000x\ dx$

11. According to L'Hospital's Rule, what is the limit of the fraction $\frac{sin(2\pi x)}{cos(\pi x)}$ as x approaches $\frac{1}{2}$?

(a) 0 (b) π (c) 2 (d) $\frac{\pi^2}{8}$ (e) 4π

12. Compute $\int_0^1 xe^x\ dx$.

(a) $e + e^2$ (b) $1 + e$ (c) $1 - e$ (d) 1 (e) e

13. Compute $\int_0^{\frac{\pi}{4}} cos^2(x)\ dx$.

(a) $-\frac{\pi}{8}$ (b) $\frac{\pi}{8}+\frac{1}{4}$ (c) 14 (d) $\pi+\frac{1}{4}$ (e) $\frac{\pi}{4}$

14. Compute

$$\int_0^{\frac{\pi}{4}} sec^4(x)tan(x)\ dx.$$

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$ (e) $\frac{3}{2}$

15. Which of the following integrals would be used to solve $\int \frac{x^3}{\sqrt{1+x^2}}\ dx$ by trigonometric substitution?

(a) $\int sin(\theta)tan(\theta)\ d\theta$
(b) $\int cos(\theta)\ d\theta$
(c) $\int sin(\theta)\ d\theta$
(d) $\int tan(\theta)\ d\theta$
(e) $\int sec(\theta)tan^3(\theta)\ d\theta$

16. Use logarithmic differentiation to compute the derivative of $y = \frac{x^5(x+7)^4}{(x+2)^3(x+5)^2}$.

(a) $y' = \frac{5}{x} - \frac{4}{x+7} + \frac{3}{x+2} - \frac{2}{x+5}$

(b) $y' = \frac{5}{x} + \frac{4}{x+7} - \frac{3}{x+2} + \frac{2}{x+5}$

(c) $y' = y(5x + 4(x+7) - 3(x+2) - 2(x+5))$

(d) $y' = y(\frac{5}{x} + \frac{4}{x+7}) - \frac{3}{x+2} - \frac{2}{x+5}$

(e) $y' = y(\frac{5}{x} + \frac{4}{x+7} - \frac{3}{x+2} - \frac{2}{x+5})$

17. How would you rewrite $\frac{6}{(x-1)(x+2)}$ in order to find its antiderivative by the method of partial fractions?

(a) $\frac{1}{x-1} - \frac{5}{x+2}$ (b) $\frac{4x}{x-1} - \frac{2x+3}{x+2}$ (c) $\frac{3}{x-1} - \frac{1}{x+2}$

(d) $\frac{2}{x-1} - \frac{2}{x+2}$ (e) $\frac{7}{x-1} - \frac{x}{x+2}$

18. Find the y-coordinate of the centroid of the region bounded by $y = cos(x)$, $y = 0$, $x = 0$ and $x = \frac{\pi}{2}$. The area of this region is 1 square unit.

(a) $\int_0^{\frac{\pi}{2}} sin^2(x)\ dx$

(b) $\frac{1}{2}\int_0^{\frac{\pi}{2}} xcos(x)\ dx$

(c) $\frac{1}{2}\int_0^{\frac{\pi}{2}} cos^2(x)\ dx$

(d) $\frac{\pi}{2}\int_0^{\frac{\pi}{2}} cos(x) - sin(x)\ dx$

(e) $\frac{1}{4}\int_0^{\frac{\pi}{4}} cos^4(x)\ dx$

19. Which of the following integrals represents the length along the curve $y = 3x^2 - 5$ from $x = 1$ to $x = 4$?

(a) $\int_1^4 \sqrt{1 + 36x^2}\ dx$

(b) $\int_1^4 \sqrt{1 + 6x}\ dx$

(c) $\int_1^4 2\pi(3x^2 - 5)\sqrt{1 + 36x^2}\ dx$

(d) $\int_1^4 2\pi(3x^2 - 5)\sqrt{1 + 6x}\ dx$

(e) $\int_1^4 \sqrt{1 - 6x^2}\ dx$

20. Find the surface area generated by rotating the curve $y = (x - 2)^{\frac{3}{2}}$ between $x = 2$ and $x = 5$ around the x-axis.

(a) $\int_2^5 \sqrt{1 + \frac{9}{4}(x - 2)}\ dx$

(b) $\int_2^5 2\pi(x - 2)^{\frac{3}{2}}\sqrt{1 + \frac{9}{4}(x - 2)}\ dx$

(c) $\int_2^5 2\pi(x - 2)^{\frac{3}{2}}\sqrt{1 + (x - 5)}\ dx$

(d) $\int_2^5 2\pi(x - 2)^{\frac{3}{2}}\sqrt{1 + \frac{3}{2}(x - 2)^{\frac{1}{2}}}\ dx$

(e) $\int_2^5 \sqrt{1 + \frac{3}{2}(x - 5)}\ dx$

21. If the demand function is $p = 5 - \frac{x}{40}$ and the selling price is \$4, find the consumer surplus.

(a) $\int_0^{4.9} \frac{1}{10} - \frac{x}{40}\ dx$

(b) $\int_0^{40} \frac{1}{10} - \frac{x}{40}\ dx$

(c) $\int_0^{4.9} 5 - \frac{x}{40}\ dx$

(d) $\int_0^{4.9} 1 - \frac{x}{40}\ dx$

(e) $\int_0^{40} 1 - \frac{x}{40}\ dx$

22. If you flip a fair coin five times, what is the probability of getting **at least** four heads?

(a) $\frac{3}{32}$ (b) $\frac{3}{16}$ (c) $\frac{3}{8}$ (d) $\frac{1}{32}$ (e) $\frac{1}{8}$

23. An unfair coin has a sixty percent chance of coming up tails (q=$\frac{3}{5}$). In flipping the coin five times, what is the chance of getting exactly four heads?

(a) $10(\frac{2}{5})^3(\frac{3}{5})^2$ (b) $10(\frac{2}{5})^4(\frac{3}{5})$ (c) $5(\frac{2}{5})^4(\frac{3}{5})$

(d) $6(\frac{2}{5})^3(\frac{3}{5})$ (e) $5(\frac{3}{5})^4(\frac{2}{5})$

24. Suppose $f(0) = f'(0) = f''(0) = 4$, $f^{(3)}(0) = 12$ and $f^{(4)}(0) = 72$. Which of the following is the fourth Taylor polynomial for $f(x)$?

(a) $4 + 4x + 2x^2 + 2x^3 + 3x^4$

(b) $4 + 2x - x^2 + 2x^3 + 3x^4$

(c) $4 - 4x + 2x^2 - 2x^3 + 3x^4$

(d) $1 + 4x + x^2 + x^3 + x^4$

(e) $4 - 4x + 2x^2 - x^3 + x^4$

25. Suppose that the second Taylor polynomial for $f(x)$ is $P_2(x) = 3 + x + 5x^2$. What approximation for $f(.1)$ does this give?

(a) 5.13 (b) 325 (c) 1.35 (d) 3.15 (e) 2.513

26. Give an upper bound on the error in approximating e^{-1} by $P_4(-1) = \frac{9}{24}$ by applying Taylor's Theorem.

(a) $\frac{e}{3(4!)}$ (b) $\frac{1}{3(5!)}$ (c) $\frac{e}{6!}$ (d) $\frac{1}{8!}$ (e) $\frac{1}{5!}$

27. What integral represents the hydrostatic force on the end of a rectangular tub of width 2m and depth 5m filled with water?

(a) $9800 \int_0^5 5x\ dx$ (b) $9800 \int_0^2 5x\ dx$ (c) $9800 \int_0^5 2x\ dx$

(d) $9800 \int_0^2 2x\ dx$ (e) $9800 \int_0^5 x^2\ dx$

28. Find a Taylor Series for the derivative of $sinh(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots$.

(a) $1 + \frac{x^4}{4!} + \frac{x^8}{8!} + \ldots$

(b) $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots$

(c) $\frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \ldots$

(d) $\frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \ldots$

(e) $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots$

29. Give a Taylor Series expansion for $tan^{-1}(x)$ if the Taylor Series expansion for $\frac{1}{1+x^2} = \sum_{k=0}^{\infty}(-1)^k x^{2k}$.

(a) $\sum_{k=1}^{\infty}(-1)^k x^{2k+1}$

(b) $\sum_{k=0}^{\infty}(-1)^k x^{2k+1}$

(c) $\sum_{k=0}^{\infty}(-1)^{k+1} \frac{x^{2k}}{k+1}$

(d) $\sum_{k=0}^{\infty}(-1)^k \frac{x^{2k+1}}{2k+1}$

(e) $\sum_{k=0}^{\infty}(-1)^k (2k) x^{2k-1}$

30. In class, we found a Taylor Series expansion for $\int e^{-x^2}\, dx$ by integrating the expansion for e^u with $u = -x^2$. What is the Radius of Convergence of the resulting Taylor Series expansion?

(a) ∞ (b) 10 (c) 2 (d) 1 (e) 0