

**Math 120 – Calculus B**  
**Fall Semester 2000**  
**First Midterm**  
**Thursday, September 21**

Name: \_\_\_\_\_ Section: \_\_\_\_\_

This examination consists of 15 problems, worth a total of 100 points on 8 pages, including this front cover. If problems are missing from your copy, you must ask for a new copy right away. The first ten problems are multiple choice **with no partial credit for any reason**. Be sure to indicate your single answer to each question by placing an  $\times$  through that letter on the answer grid below. Students will **NOT** be allowed extra time to fill in the grid after the exam has ended if they forget to do so during the exam!

1.  a  b  c  d  e

6.  a  b  c  d  e

2.  a  b  c  d  e

7.  a  b  c  d  e

3.  a  b  c  d  e

8.  a  b  c  d  e

4.  a  b  c  d  e

9.  a  b  c  d  e

5.  a  b  c  d  e

10.  a  b  c  d  e

The last 5 problems are partial credit problems. Be sure to show all work legibly. Clearly indicate your final answer, which should be simplified whenever possible. **Books, notes and CALCULATORS are NOT permitted. Sign the following honor code statement:**

“On my honor, I have neither given nor received unauthorized aid on this exam.”

\_\_\_\_\_

1. Which of the following expresses  $\int_2^5 x^4 dx$  as a limit of Riemann sums?

(a)  $\lim_{n \rightarrow \infty} \sum_{i=1}^n (5 + \frac{3i}{2})^4 \frac{3}{n}$       (b)  $\lim_{n \rightarrow \infty} \sum_{i=1}^n (2 + \frac{3i}{n})^5 \frac{5}{n}$

(c)  $\lim_{n \rightarrow \infty} \sum_{i=1}^n (1 + \frac{3i}{n})^4 \frac{3}{n}$       (d)  $\lim_{n \rightarrow \infty} \sum_{i=1}^n (2 + \frac{3i}{n})^4 \frac{3}{n}$

(e)  $\lim_{n \rightarrow \infty} \sum_{i=1}^n (2 + \frac{3i}{4})^5 \frac{3}{5n}$

2. Which of the following is an expression for  $\int_1^2 x dx$  directly from the definition? (Recall that  $\sum_{i=1}^n i = \frac{n(n+1)}{2}$ .)

(a)  $\lim_{n \rightarrow \infty} (\frac{2}{n} + \frac{n(n+1)}{2n^2})$       (b)  $\lim_{n \rightarrow \infty} (1 + \frac{n(n+1)}{2n^2})$

(c)  $\lim_{n \rightarrow \infty} (\frac{1}{2} + \frac{n(n+1)}{n^2})$       (d)  $\lim_{n \rightarrow \infty} (n + \frac{n(n+1)}{2n})$

(e)  $\lim_{n \rightarrow \infty} (\frac{1}{n} + \frac{n(n+1)}{2n^2})$

3. Suppose  $x^3 \leq f(x) \leq x^2$  for  $0 \leq x \leq 1$ . Which of the following must be true?

(a)  $\frac{1}{3} \leq \int_0^1 f(x) dx \leq \frac{1}{2}$

(b)  $1 \leq \int_0^1 f(x) dx \leq 3$

(c)  $\frac{1}{4} \leq \int_0^1 f(x) dx \leq \frac{1}{3}$

(d)  $\int_0^1 f(x) dx$  is negative

(e)  $\int_0^1 f(x) dx$  is zero

4. Compute

$$\int_2^3 3x^2 - 2x + 7 \, dx$$

- (a) 59      (b) 14      (c) -18      (d) 39      (e) 21

5. Compute

$$\int \sec^2(x)\tan(x) \, dx$$

- (a)  $\tan(\sec(x)) + C$     (b)  $\sec(\tan(x)) + C$     (c)  $\sec^2(x)$   
(d)  $\frac{1}{2}\sec(x)\tan(x) + C$       (e)  $\frac{1}{2}\tan^2(x) + C$

6. Find the area between  $y = 1$ ,  $y = -2$ ,  $x = y^2$  and  $x = 2y - 2$ .

- (a)  $\frac{8}{3}$       (b) 8      (c) 12      (d)  $-\frac{8}{3}$       (e) -12

7. The natural length of a spring is 30 cm. If a 10 N force holds it at 35 cm, how much work was required to move it there from its natural position?

- (a) .5 J    (b) 50 J    (c) 25 J    (d) .25 J    (e) 5 J

8. Find the average value of the function  $y = \cos\left(\frac{\pi}{2}\sin(x)\right)\cos(x)$  over the interval between 0 and  $\frac{\pi}{2}$ .

- (a)  $\frac{4}{\pi^2}$     (b)  $\frac{\pi^2}{8}$     (c)  $\frac{8}{\pi^2}$     (d)  $\frac{\pi}{4}$     (e)  $\frac{\pi}{8}$

9. Which of the following is the derivative of  $g(x) = e^{\tan(x)}$ ?

- (a)  $\tan(x) e^{\sec^2(x)}$     (b)  $\sec^2(x) e^{\tan(x)}$     (c)  $e^{\tan(x)} + C$   
(d)  $e^{\sec^2(x)}$     (e)  $e^{\sec(x)\tan(x)}$

10. Integrate

$$\int_0^{\frac{\pi}{2}} 2\sin(x)\cos(x)e^{\sin^2(x)} dx$$

- (a)  $e - 1$    (b)  $1 - e$    (c)  $e^2 - 1$    (d)  $\frac{e^2}{2} - \frac{1}{2}$    (e)  $e$

- 11.** Approximate the area between  $y = x$  and  $y = x^2$  using left endpoints and  $n = 3$  subintervals.

- 12.** If

$$h(x) = \int_{2-4x^2}^{4x^2-2} \sin(t^2) dt$$

compute  $h'(x)$ .

- 13.** Find the volume of the solid formed by rotating the region between  $x = 1$ ,  $y = 0$  and  $x = y^2$  around the line  $x = 1$ .

**14.** Compute the volume of the solid formed by rotating the area under the curve  $y = \sin(x^2)$  between 0 and  $\sqrt{\pi}$  around the  $y$ -axis.

**15.** If a child succeeds in emptying a rectangular bathtub by splashing all of the water over the sides, give a formula for the work done by the child in terms of the length ( $l$ ), width ( $w$ ) and height ( $h$ ) of the tub. (Using metric units the weight of water is  $9800 \frac{N}{m^3}$ .) Your work should include at least one Riemann sum approximating the work.