Multiple Choice

This test consists of 12 multiple-choice questions and four partial credit questions. The correct answer to each multiple choice question is (a).

1. (5 pts.) The area of the region bounded by the curves $y = |x|$ and $y = x^2$ is

(a) $\frac{1}{3}$ (b) $\frac{1}{7}$ (c) 0 (d) 1 (e) 2

2. (5 pts.) Use the disk method (in other words, *not* the method of cylindrical shells) to set up an integral which gives the volume of the solid obtained by revolving the region bounded by $y = x$ and $y = \sqrt{x}$ about the line $y = -2$. The resulting integral is

(a) $\pi \int_0^1 [(2+\sqrt{x})^2 - (2+x)^2]\,dx$ (b) $\pi \int_0^1 [(2+y)^2 - (2+y^2)^2]\,dy$

(c) $\pi \int_0^1 [(2-x)^2 - (2-\sqrt{x})^2]\,dx$ (d) $\pi \int_0^1 [(x^2+2) - (x-2)]\,dx$

(e) $\pi \int_0^1 [(x^2+2) - (x-2)]\,dx$

3. (5 pts.) Use the method of cylindrical shells to set up an integral which gives the volume of a sphere of radius r with center at the origin. The resulting integral is

(a) $4\pi \int_0^r x\sqrt{r^2 - x^2}\,dx$ (b) $4\pi \int_0^r x(r^2 - x^2)\,dx$

(c) $4\pi \int_0^r x^2\sqrt{r^2 - x^2}\,dx$ (d) $4\pi \int_0^r (r^2 - x^2)\,dx$

(e) $4\pi \int_0^r (r-x)\sqrt{r^2 - x^2}\,dx$

4. (5 pts.) Let $f(x) = 4 - 3x^2, 0 \le x \le 2$. Find the number c in $[0, 2]$ such that $f(c)$ is equal to the average of f on the interval $[0, 2]$. The number c is

(a) $\frac{2}{\sqrt{3}}$ (b) 1 (c) 2 (d) $\frac{1}{2}$ (e) 0

5. (5 pts.) Suppose that g is the inverse of a differentiable function f. Assume that $f(-1) = 0$ and that $f'(-1) = -2$. What is the number $g'(0)$?

(a) $-\frac{1}{2}$ (b) 2 (c) 0 (d) -2 (e) -1

6. (5 pts.) If $f(x) = \sqrt{2+x}$, find a formula for the inverse function $f^{-1}(x)$.

(a) $x^2 - 2$ (b) $\dfrac{x^2}{2}$ (c) $\sqrt{x^2-2}$ (d) $x - 2$ (e) $(x-2)^2$

7. (5 pts.) If $y = e^{e^x}$ then $\dfrac{dy}{dx} =$

(a) $e^{e^x} e^x$ (b) e^{e^x} (c) $e^{e^{x-1}}$ (d) $e^x e^{e^{x-1}}$ (e) $e^x e^{x-1}$

8. (5 pts.) If $f(x) = \ln \dfrac{x}{x-1}$ then $f'(x) =$

(a) $\dfrac{1}{x} - \dfrac{1}{x-1}$ (b) $\dfrac{1}{x} + \dfrac{1}{x-1}$ (c) $\dfrac{x-1}{x}$ (d) $\dfrac{x}{x-1}$ (e) $\dfrac{1}{x} - 1$

9. (5 pts.) Simplify the expression $\ln 3 + \dfrac{1}{3}\ln 8$.

(a) $\ln 6$ (b) $\dfrac{1}{3}\ln 11$ (c) $\dfrac{1}{3}\ln 26$ (d) 1 (e) $\ln 2$

10. (5 pts.) Evaluate the indefinite integral $\int e^x \cos e^x \, dx$.

(a) $\sin e^x + C$ (b) $e^x \sin e^x + C$ (c) $\cos (e^x)^2 + C$

(d) $\dfrac{(e^x)^2 \cos x}{2} + C$ (e) $e^{\sin x} + C$

11. (5 pts.) Evalute the limit $\lim_{x\to\infty} \dfrac{e^x}{1+e^{2x}}$.

(a) 0 (b) 1 (c) ∞ (d) 2 (e) $\dfrac{1}{2}$

12. (5 pts.) A large spring is extended one foot beyond its natural length, and measurements show that a force of 6 lb is required to hold it in that position. The spring is then released and it returns to its natural length. How much *work* would now be done in stretching the spring 6 inches? (Remember that there are 12 inches in a foot.)

(a) $\dfrac{3}{4}$ ft-lb (b) 3 ft-lb (c) $\dfrac{9}{2}$ ft-lb (d) 12 ft-lb (e) 35 ft-lb

Partial Credit

13. (10 pts.) Set up an integral which gives the volume of a right circular cone with height 5 and base radius 4.

14. (10 pts.) A cylindrical tank stands upright with its circular base on the ground. It has a base radius 1 meter and is 2 meters high, and it is filled to the brim with water. Find the work done in pumping half of the water in the tank up to the top. (The density of water is $1000\,kg/m^3$.)

15. (10 pts.) Sketch the region bounded by $y = \dfrac{1}{x^2}, y = 0, x = 1, x = 3$ and find the volume of the solid obtained by revolving the region about the line $y = -1$.

16. (10 pts.) Find the equation of the line tangent to the graph of the function $y = \dfrac{e^x}{x}$ at the point $(1, e)$.