Multiple Choice

This test consists of 12 multiple-choice questions and four partial credit questions. The correct answer to each multiple choice question is (a).

1. (5 pts.) Find the slope of the tangent line to the curve $y = \tan^{-1}(3x^2)$ at $x = 1$.

(a) $\frac{3}{5}$ (b) 2 (c) $\frac{2}{3}$ (d) $\frac{\pi}{3}$ (e) $\frac{1}{2}$

2. (5 pts.) Calculate $\int_1^2 \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)\sqrt{x}\,dx$.

(a) $\frac{5}{2}$ (b) $2^{3/2} - 1$ (c) $\frac{1}{2}$ (d) $\frac{\pi}{4}$ (e) 2

3. (5 pts.) Which of the following equations describes the partial fraction decomposition of a rational function of the form $\dfrac{x^2+2x+3}{(x-1)^2(x-2)(x^2+4)}$?

(a) $\dfrac{x^2+2x+3}{(x-1)^2(x-2)(x^2+4)} = \dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{x-2} + \dfrac{Dx+E}{x^2+4}$

(b) $\dfrac{x^2+2x+3}{(x-1)^2(x-2)(x^2+4)} = \dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{x-2} + \dfrac{D}{x^2+4}$

(c) $\dfrac{x^2+2x+3}{(x-1)^2(x-2)(x^2+4)} = \dfrac{A}{x-1} + \dfrac{Bx+C}{(x-1)^2} + \dfrac{D}{x-2} + \dfrac{E}{x^2+4}$

(d) $\dfrac{x^2+2x+3}{(x-1)^2(x-2)(x^2+4)} = \dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{x-2} + \dfrac{Dx}{x^2+4}$

(e) $\dfrac{x^2+2x+3}{(x-1)^2(x-2)(x^2+4)} = \dfrac{A}{x-1} + \dfrac{Bx}{(x-1)^2} + \dfrac{C}{x-2} + \dfrac{Dx+E}{x^2+4}$

4. (5 pts.) According to the method of partial fractions, there is an equation of the form

$$\frac{x}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

for some numbers A, B, and C. What is the number B?

(a) -2 (b) $\frac{1}{2}$ (c) -1 (d) 3 (e) 0

5. (5 pts.) Evaluate $\displaystyle\int_1^2 t^2 \ln(t)\, dt$.

(a) $\dfrac{8}{3}\ln(2) - \dfrac{7}{9}$ (b) $4\ln(2) + 1$ (c) $\dfrac{9}{2}\ln(2) - \dfrac{7}{2}$

(d) $3\ln(2) + \dfrac{2}{9}$ (e) $4(\ln 2)^2$

6. (5 pts.) Suppose that $x = 3\tan\theta$. Find an expression for $\cos\theta$ in terms of x. (Hint: draw a triangle.)

(a) $\dfrac{3}{\sqrt{9+x^2}}$ (b) $\dfrac{\sqrt{x^2-9}}{x}$ (c) $\dfrac{3}{\sqrt{9-x^2}}$ (d) $\dfrac{\sqrt{9+x^2}}{3x}$ (e) $\dfrac{\sqrt{9-x^2}}{\sqrt{9+x^2}}$

7. (5 pts.) Calculate $\displaystyle\int_0^{\pi/2} \sin^2 x \cos^5 x\, dx$.

(a) $\dfrac{1}{3} - \dfrac{2}{5} + \dfrac{1}{7}$ (b) $\dfrac{1}{6}$ (c) $\dfrac{1}{5} + \dfrac{4}{7} - \dfrac{1}{9}$

(d) $1 - \dfrac{2}{3} + \dfrac{1}{5}$ (e) 0

8. (5 pts.) Evaluate the indefinite integral $\displaystyle\int x\cos 2x\, dx$.

(a) $\dfrac{x\sin 2x}{2} + \dfrac{\cos 2x}{4} + C$ (b) $\dfrac{x\sin 2x}{2} + \dfrac{\cos 4x}{2} + C$

(c) $\dfrac{x^2\cos 2x}{2} - \dfrac{x^2\sin 2x}{4} + C$ (d) $\dfrac{x^2\sin 2x}{4} + C$

(e) $\sin x^2 + C$

9. (5 pts.) Calculate $\displaystyle\lim_{x\to\infty} \sin(\tan^{-1} x)$.

(a) 1 (b) 0 (c) ∞ (d) $-\infty$ (e) 2

10. (5 pts.) Consider the integral $\int \frac{x^5}{\sqrt{x^2-9}}\,dx$. Which of the following trigonometric substitutions would eliminate the square root from this integral?

(a) $x = 3\sec\theta$ (b) $x = 3\cos\theta$ (c) $x = 3\tan\theta$

(d) $x = 3\cot\theta$ (e) $x = 3\sin\theta$

11. (5 pts.) What is $\sin^{-1}(\frac{\sqrt{3}}{2})$?

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) π (e) $\frac{1}{2}$

12. (5 pts.) Calculate $\lim_{x\to 0} \frac{\sin^{-1} x}{2x}$

(a) $\frac{1}{2}$ (b) 0 (c) ∞ (d) -2 (e) $-\infty$

Partial Credit

13. (10 pts.) Calculate $\displaystyle\int \frac{2x^2+1}{x(x^2+1)}\,dx$.

14. (10 pts.) Find the length of the curve $y = \ln(\cos x)$ for $0 \le x \le \dfrac{\pi}{4}$.

15. (10 pts.) Calculate $\displaystyle\int \frac{dx}{(x^2+1)\sqrt{x^2+1}}$

16. (10 pts.) Calculate

$$\lim_{x\to 0} \frac{\cos x - 1 + (x^2/2)}{x^4}$$

(Hint: don't give up too soon!)