Multiple Choice

This test consists of 12 multiple-choice questions and four partial credit questions. The correct answer to each multiple choice question is (a).

1. (5 pts.) If A dollars are invested at an annual rate r compounded monthly, how much money will be in the investment account after 2 years?

(a) $A(1+\frac{r}{12})^{24}$ (b) $A(1+\frac{r}{6})^{12}$ (c) $A(1+r)^2$

(d) $(A+\frac{r}{12})^{12}$ (e) Ae^{24r}

2. (5 pts.) For what values of r is the function $y = e^{rx}$ a solution of the differential equation $y'' - 3y' + 2y = 0$?

(a) $r = 1, 2$ (b) $r = 0, 2, -2$ (c) $r = 2, 3$

(d) $r = 0$ (e) $r = -1$

3. (5 pts.) What amount should be invested at an annual interest rate of 4%, compounded continuously, in order for the investment to reach \$100,000 in 25 years?

(a) $100,000 \cdot e^{-1}$ (b) $100,000 \cdot e$ (c) $100,000 \cdot 1.04^{-25}$

(d) $100,000 \cdot 1.04^{25}$ (e) $100,000 \cdot 25^{-1.04}$

4. (5 pts.) A bacteria culture, growing exponentially, starts with 500 bacteria. After 2 hours there are 1200 bacteria. Find the number of bacteria after 5 hours. In the answers, the notation $\exp(x)$ stands for the exponential e^x.

(a) $500 \cdot \exp\left(\frac{5}{2}\ln\frac{12}{5}\right)$ (b) $500 \cdot \exp\left(\frac{2}{5}\ln\frac{5}{12}\right)$ (c) $500 \cdot \left(\frac{12}{5}\right)^3$

(d) $500 \cdot \exp\left(\frac{5}{12}\right)$ (e) 2250

5. (5 pts.) A radioactive substance has half life of 100 years. If a sample has a mass of 500g, when will it be reduced to 20g?

(a) $\frac{100\ln 25}{\ln 2}$ years. (b) $100\ln 50$ years. (c) $1,250$ years.

(d) $\frac{100\ln 2}{\ln 5}$ years. (e) $2,500$ years.

6. (5 pts.) Find the coefficient of a^2b^7 in the expansion of $(a+b)^9$.

(a) 36 (b) 21 (c) 7 (d) 84 (e) 14

7. (5 pts.) A certain die has a shape of a triangle, with sides labelled 1,2,3. When it is rolled, each side appears on the bottom with probability of $\frac{1}{3}$. If die is rolled 4 times, what is the probability that the side labelled "2" will appear on the bottom at least 2 times?

(a) $6\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^2+4\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)+\left(\frac{1}{3}\right)^4$

(b) $\left(\frac{1}{3}\right)^4$

(c) $2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^2+3\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)+4\left(\frac{1}{3}\right)^4$

(d) $6\left(\frac{1}{3}\right)^2+4\left(\frac{1}{3}\right)^3+\left(\frac{1}{3}\right)^4$

(e) $\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^2$

8. (5 pts.) What approximation of $\cos(0.6)$ does the 2nd degree Taylor polynomial of the function $f(x)=\cos(x)$ give?

(a) 0.82 (b) 1.36 (c) 0.83 (d) 0.825 (e) 1

9. (5 pts.) Find the third degree Taylor polynomial of the function $f(x)=2\sin(x)+\cos(x)$.

(a) $1+2x-\frac{1}{2!}x^2-\frac{2}{3!}x^3$

(b) $1+x+\frac{1}{2!}x^2+\frac{1}{3!}x^3$

(c) $x-\frac{2}{3!}x^3$

(d) $2-\frac{1}{2!}x^2+\frac{1}{3!}x^3$

(e) $1+2x+\frac{1}{2!}x^2$

10. (5 pts.) Which of the following represents the kth Taylor coefficient of the function $f(x) = e^{3x} + e^x$?

(a) $\dfrac{3^k + 1}{k!}$ (b) $\dfrac{4^k}{k!}$ (c) $\dfrac{3k + 1}{(3k)!}$ (d) $\dfrac{4}{k!}$ (e) $3k + 1$

11. (5 pts.) Consider the differential equation $y' = y - 2\sin x$. Of the following two functions

(I) $y = \cos x + \sin x$

(II) $y = \cos x + e^{-x}$

which, if any, are solutions of the differential equation?

(a) (I) only

(b) (II) only

(c) (I) and (II)

(d) Neither is a solution

(e) It is impossible to determine from the given information.

12. (5 pts.) Find the centroid (center of mass) of the region bounded by $y = 1 - x^4$ and $y = 0$.

(a) $(0, \frac{4}{9})$ (b) $(0, \frac{1}{2})$ (c) $(\frac{1}{3}, 0)$ (d) $(\frac{1}{3}, \frac{8}{9})$ (e) $(\frac{1}{2}, 0)$

Partial Credit

13. (10 pts.) Represent $0.152323232323232\ldots$ as a fraction or sum of fractions.

14. (10 pts.) Use the linear approximation to the function $f(x) = \sqrt{9+x}$ to approximate $\sqrt{10}$, and estimate the error involved in making the approximation.

15. (10 pts.) In many card games each person receives a 13-card hand dealt at random from a 52-card deck. You should give the answers to the following questions in terms of binomial coefficients. *Don't attempt to express the binomial coefficients as single integers!* The numbers involved may turn out to be very large.

(a) How many different 13-card hands are possible?

(b) A regular deck contains 16 face cards (4 in each suit). How many 13 card hands are there which contain *no* face cards?

(c) If you are dealt a 13-card hand from a shuffled deck, what is the probability of not getting any face cards?

16. (10 pts.) Suppose that $|5y| < 1$. Calculate the sum

$$\sum_{k=1}^{\infty} (5y)^k \, .$$