Name: _____

Instructor: _____

Math 120, Final

May 7, 2001

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- Be sure that you have all 25 questions (6 points each).
- The exam lasts for two hours.
- You are to hand in just the front page.

Good Luck!

Please mark your answers with an **X**! Do NOT circle them!

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Score: _____

1.(6 pts.) The area of the region enclosed by the curves $y = x+1$ and $y = (x-1)^2$ between $x = 0$ and $x = 2$ is

(a) $\frac{26}{3}$ (b) $\frac{-10}{3}$ (c) $\frac{10}{3}$ (d) $\frac{14}{3}$ (e) $\frac{9}{2}$

2.(6 pts.) Let R be the region between the graph of $y = \frac{1}{x^2}$ and the x-axis for x between 1 and 3. Consider the solid obtained by rotating R about the line $y = -1$. Which of the following integrals gives the volume of this solid?

(a) $\pi \int_1^3 \frac{1}{x^4} - 1 \, dx$

(b) $\pi \int_1^3 \left(\frac{1}{x^2} + 1\right)^2 - 1 \, dx$

(c) $\pi \int_1^3 \left(\frac{1}{x^2} + 1\right)^2 + 2 \, dx$

(d) $\pi \int_1^3 \frac{1}{x^4} \, dx$

(e) $\pi \int_1^3 \frac{1}{y} \, dy$

3.(6 pts.) Let R be the region between the graph of $y = \sin x$ and the x-axis for x between 0 and π. Consider the solid obtained by rotating R about the line $x = -1$. Which of the following integrals gives the volume of this solid?

(a) $\displaystyle\int_0^\pi 2\pi(x+1)\sin x\,dx$

(b) $\displaystyle\int_{-1}^\pi \pi\sin^2 x\,dx$

(c) $\displaystyle\int_{-1}^\pi 2\pi x\sin x\,dx$

(d) $\displaystyle\int_0^\pi \pi(x\sin x)^2\,dx$

(e) $\displaystyle\int_0^\pi 2\pi x(1-\sin x)\,dx$

4.(6 pts.) The work done in stretching a large spring from its natural length of 2 meters to 5 meters equals 6 joules. Find the work done in stretching it from 5 meters to 8 meters.

(a) 36 joules (b) 3 joules (c) 6 joules (d) 18 joules (e) 2 joules

5.(6 pts.) The average value of the function $f(x) = 3 - 3x^2$ on the interval $[0, 2]$ is

(a) 1 (b) -4 (c) 0 (d) 2 (e) -1

6.(6 pts.) Let

$$f(x) = \sqrt{x^3 + x^2 + x + 1}$$

for $x \geq 0$, and let g be the inverse function for f. Observe that $f(1) = 2$. What is $g'(2)$?

(a) $\frac{3}{2}$ (b) $\frac{1}{3}$ (c) 4 (d) $\frac{1}{2}$ (e) $\frac{2}{3}$

7.(6 pts.) The limit

$$\lim_{x \to 2^-} \ln(2 - x)$$

is

(a) 1 (b) ∞ (c) 0 (d) $-\infty$ (e) -1

8.(6 pts.) Solve the equation

$$\ln\left(\frac{x+1}{x-1}\right) = 2$$

(a) $x = \dfrac{e^2+1}{e^2-1}$ (b) $x = \dfrac{e^2}{2}$ (c) $x = \dfrac{\ln 2}{1 - \ln 2}$

(d) $x = \dfrac{e-1}{e+1}$ (e) $x = \dfrac{2}{e^2}$

9.(6 pts.) The equation of the line tangent to the curve $y = x\ln(x)$ at the point (e, e) is

(a) $y = e$ (b) $y = ex - 1$ (c) $y = 2x + 1$

(d) $y = x + 2$ (e) $y = 2x - e$

10.(6 pts.) Find the derivative of $x^{\sin x}$.

(a) $x^{\sin x}\cos x$ (b) $x^{\cos x}$

(c) $x^{\sin x}(\ln x)(\cos x)$ (d) $(\cos x \ln x + \frac{1}{x}\sin x)x^{\sin x}$

(e) $\dfrac{x^{\sin x}\cos x}{x}$

11.(6 pts.) Calculate $\displaystyle\int_e^{e^2} \frac{dx}{x\ln(x)}$.

(a) 3 (b) $\ln 2$ (c) $e/2-1$ (d) $1/2$ (e) $1-\ln 2$

12.(6 pts.) Find $\displaystyle\lim_{x\to\infty} \frac{x}{\ln(1+e^{2x})}$, if it exists.

(a) $1/2$

(b) limit does not exist

(c) $1/4$

(d) 0

(e) 1

13.(6 pts.) Evaluate the integral $\int_1^3 x^2 \ln x \, dx$.

(a) $9\ln 3 - \frac{79}{9}$ (b) $9\ln 3 - \frac{82}{9}$ (c) $9\ln 3 - 9$

(d) $9\ln 3 - 3$ (e) $9\ln 3 - \frac{26}{9}$

14.(6 pts.) Calculate $\int_0^{\pi/2} \sin^6 x \cos^3 x \, dx$.

(a) $2/63$ (b) $1/37$ (c) $3/127$ (d) $1/32$ (e) $\pi/130$

15.(6 pts.) In order to evaluate the integral $\displaystyle\int \frac{dx}{x^2+9}$, what substitution should you make?

(a) $x = 3\tan u$ (b) $u = x^2$ (c) $u = 3 + \tan x$

(d) $x = \dfrac{\tan u}{3}$ (e) $u = 3\tan^{-1} x$

16.(6 pts.) A bank account pays 6% interest, compounded continuously. If you deposit \$1,000 into the account, how long will it be before the account grows to \$2,000?

(a) $\dfrac{\ln 2}{0.06} \approx 11.5$ years (b) $\dfrac{\ln 2}{e^{0.06}-1} \approx 11.2$ years

(c) $\dfrac{1}{0.12} \approx 8.3$ years (d) $\dfrac{1}{0.06} \approx 16.66$ years

(e) $\dfrac{e^{0.06}}{0.06} \approx 17.7$ years

17.(6 pts.) Which of the following is the correct partial fraction representation of

$$\frac{2x^3 - x + 1}{(x-2)^3(x^2+1)^2(x^2-1)}$$

(a) $\dfrac{A}{x-2} + \dfrac{B}{(x-2)^2} + \dfrac{C}{(x-2)^3} + \dfrac{D}{x^2+1} + \dfrac{F}{(x^2+1)^2} + \dfrac{H}{x^2-1}$

(b) $\dfrac{A}{(x-2)^3} + \dfrac{B}{x^2+1} + \dfrac{C}{x^2-1}$

(c) $\dfrac{A}{x-2} + \dfrac{B}{(x-2)^2} + \dfrac{C}{(x-2)^3} + \dfrac{Dx+E}{(x^2+1)^2} + \dfrac{Fx+G}{x^2-1}$

(d) $\dfrac{A}{x-2} + \dfrac{B}{(x-2)^2} + \dfrac{C}{(x-2)^3} + \dfrac{Dx+E}{x^2+1} + \dfrac{Fx+G}{(x^2+1)^2} + \dfrac{H}{x-1} + \dfrac{I}{x+1}$

(e) $\dfrac{A}{x-2} + \dfrac{B}{(x-2)^2} + \dfrac{C}{(x-2)^3} + \dfrac{D}{x^2+1} + \dfrac{E}{(x^2+1)^2} + \dfrac{H}{x-1} + \dfrac{I}{x+1}$

18.(6 pts.) Suppose that the function $f(x)$ is given by the formula

$$f(x) = \int_0^x \sqrt{t^2 + 2t}\, dt$$

Find the length of the graph of $f(x)$ for x between 0 and 2.

(a) 1 (b) $\frac{3}{2}$ (c) 2 (d) 3 (e) 4

19.(6 pts.) Find the centroid of the region above the x-axis and below the graph of $y = x^2$ for x between -1 and 1.

(a) $(0, 3/2)$ (b) $(0, 3/10)$ (c) $(0, 2/15)$ (d) $(0, 2/3)$ (e) $(0, 4/5)$

20.(6 pts.) For which real numbers A and B is the function

$$y = Ae^{-x} + Bxe^{-x}$$

a solution to the differential equation $y'' + 2y' + y = 0$?

(a) For **no** real numbers A and B

(b) Only for positive real numbers A and B

(c) Only for $A = 0$ and $B = 1$

(d) For all real numbers A and B

(e) Only for negative real numbers A and B

21.(6 pts.) What number results from substituting $x = 1$ into the **second degree** Taylor polynomial of $f(x) = (1 + x)^{1/3}$?

(a) $\frac{2}{3}$ (b) $\frac{14}{9}$ (c) $\frac{11}{9}$ (d) $\frac{5}{3}$ (e) 2

22.(6 pts.) Calculate the sum

$$\sum_{k=0}^{10} \binom{10}{k} 2^k$$

(a) e^{20} (b) $\binom{20}{10}$ (c) 3^{10} (d) 2^{11} (e) π^{10}

23.(6 pts.) An ordinary cubical die has six sides, numbered one to six. Suppose that you throw a fair die 12 times in a row, and record for each throw the number which comes out on top. What is the probability of having exactly two fours appear in this list of twelve numbers?

(a) $\left(\frac{5}{6}\right)^{10}$

(b) $\binom{12}{2}\left(\frac{1}{6}\right)^{2}\left(\frac{5}{6}\right)^{10}$

(c) $\binom{12}{2}\left(\left(\frac{5}{6}\right)^{10}+\left(\frac{1}{6}\right)^{2}\right)$

(d) $\binom{12}{2}\binom{6}{4}$

(e) $\binom{2}{1}\binom{10}{5}$

24.(6 pts.) The infinite sum

$$\sum_{k=1}^{\infty}\frac{3^k}{k!}=3+\frac{9}{2!}+\frac{27}{3!}+\frac{81}{4!}+\cdots$$

is equal to

(a) $\frac{1}{2}$

(b) $3e$

(c) e^3-1

(d) $3e^2$

(e) $\sin 3+\cos 3$

25.(6 pts.) Which of the following power series is the Taylor series of the function $f(x) = x^2 \sin 2x$?

(a) $\displaystyle\sum_{k=0}^{\infty}(-1)^{2k+1}\frac{2^k x^k}{k!}$

(b) $\displaystyle\sum_{k=0}^{\infty}(-1)^{k}\frac{2^{2k+3} x^{2k+1}}{(2k+3)!}$

(c) $\displaystyle\sum_{k=0}^{\infty}(-1)^{k}\frac{2^{2k+1} x^{2k+3}}{(2k+1)!}$

(d) $\displaystyle\sum_{k=0}^{\infty}(-1)^{k}\frac{2^{2k+3} x^{2k+3}}{(2k+1)!}$

(e) $\displaystyle\sum_{k=0}^{\infty}(-1)^{k}\frac{2^{2k+3} x^{2k+3}}{(2k+3)!}$

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