Brief Article

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MATH 120
HOMEWORK
Assignment 3
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Let $x_1 = 1,\ x_2 = 2,\ x_3 = 2 + \varepsilon,\ \int(x) = x^2$

Find pol. of degree 1 $p(x) = mx + c$

such that $max\,\{|f(x_i) - p(x_i)| : i = 1, 2, 3,\}$ is minimal possible.

<u>Solution</u>

$p(x_1) = m + c + B = 1 = f(1)$

$p(x_2) = 2m + c - B = 4 = f(2)$

$p(x_3) - 2m + \varepsilon m + c + B = (2 + \varepsilon)^2 = 4 + 4\varepsilon + \varepsilon^2$

$f(x) = x^2$

$x_1 = 1 \qquad\qquad x_2 = 1 + \varepsilon$

$c + B = 1$

$c - B = (1 + \varepsilon)^2$

$2C = 1 + (1 + \varepsilon)^2$

$C = \frac{1+(1+\varepsilon)^2}{2}$

$2B = 1 - (1+\varepsilon)^2$

$\Rightarrow B = \frac{1-(1+\varepsilon)^2}{2}$

Problem 1.

Given $\varepsilon > 0, x_1 = 1, x_2 = 1+\varepsilon, f(x) = x^2$, find a constant C such that $max \{|f(x_i) - C|, |f(x_2) - C|\}$ is minimal possible. What is minimal error? What happens when $\varepsilon \to 0$?

Problem 2.

Let a be a real parameter. Consider the following optimization problem:

$$x_1 + x_2 \to min$$
$$x_1^2 + 3x_2^2 \leqslant a$$

Find the global minimizer for this problem as a function of the parameter a.

Problem 3.

Describe normal cone $N_{\mathbb{R}^n_+}(x)$ for any point $x \epsilon \mathbb{R}^1_+$ (positive orthant). Using this description write down necessary and sufficient optimality conditions for the problem

$f(x) \to min$

$x \epsilon \mathbb{R}^1_+,$

where f is a convex Gâteaux differentiable function.

Problem 4.

Let A, B be convex subsets in E such that $A \cap B = \oslash$. Prove that there exists $a \epsilon E, a \neq, 0$ such that

$$inf < a, x > Let A, B be convex subsets in$$

E$such that$A $\cap B = \oslash$ Let A, B be convex subsets in E such that $A \cap B = \oslash$