

MATH 120
FINAL EXAM
December 17, 2002

1. Find $(f^{-1})'(a)$ for $f(x) = \sqrt{x^3 + 2x^2 + 1}$, $a = 2$

A) 2

B) 1

C) $1/2$

D) 0

E) -1

2. Find $f''(0)$ of $f(x) = x^2e^x$

A) 0

B) 1

C) 2

D) 3

E) 4

3. Differentiate the function $f(x) = e^{\cos 5x}$

- A) $5e^{\cos 5x} \sin 5x$ B) $5e^{\cos 5x} \cos 5x$ C) $5e^{\sin 5x} \cos 5x$ D) $-5e^{\cos 5x} \sin 5x$ E)
 $-5e^{\cos 5x} \cos 5x$

4. Solve the equation for x

$$e^{x^2+2x} = 1$$

- A) 0, 2 B) 1, 2 C) ± 1 D) 0, -2 E) -1, 2

5. Find $\cos(\sin^{-1}(\frac{\sqrt{3}}{2}))$

A) $\frac{\sqrt{3}}{2}$

B) 1

C) $\frac{1}{2}$

D) 0

E) 2

6. Evaluate $\lim_{x \rightarrow -\infty} \frac{\sinh x}{e^{-x}}$

- A) 1 B) $\frac{1}{2}$ C) 0 D) -1 E) 0

7. Find the limit $\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2}$

A) 1

B) $\frac{1}{2}$

C) ∞

D)

E) 0

8. Evaluate $\int \ln x dx$

A) $x \ln x - x + C$

B) $\ln x + C$

C) $\frac{\ln x}{x} + x + C$

D) $\frac{\ln x}{x} - x + C$

E) $x \ln x + x + C$

9. Evaluate $\int xe^x dx$

A) $e^x + C$

B) $xe^x + C$

C) $xe^x - e^x + C$

D) $xe^x + e^x + C$

E) 0

10. Find the value of the integral $I = \int_0^\pi (\sin x)^3 dx$

A) 1

B) $\frac{4}{3}$

C) 0

D) 8π

E) $\frac{2\pi}{3}$

11. Find the value of the integral $I = \int_0^1 \sqrt{x^2 + 1} dx$

A) 1

B) $\ln(\sqrt{2} - 1)$

C) $\ln(\sqrt{2} + 1)$

D) $\ln 2$

E) 0

12. Find the value of the integral $I = \int_0^{\frac{1}{2}} \frac{x}{x^2-1} dx$

A) $\frac{1}{2} \ln\left(\frac{3}{4}\right)$

B) $\ln\left(\frac{3}{4}\right)$

C) $\frac{1}{2} \ln\left(\frac{4}{3}\right)$

D) $\frac{1}{2} \ln\left(\frac{2}{3}\right)$

E) $\frac{1}{2} \ln 2$

13. Solve the differential equation $y' = y$ subject to the initial condition $y(0) = 0$. From your solution find $y(e)$.

A) e^e

B) e

C) $e^e - 1$

D) 0

E) $e^e - e$

14. Find power series representation for the following function and determine the interval on which it is valid.

$$f(x) = \sin(x^2)$$

A) $\sum_{k=0}^{\infty} (-1)^k \frac{x^{4k+2}}{(2k+1)!}$, valid for all x .

B) $\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$, valid for all x .

C) $\sum_{k=0}^{\infty} \frac{(-1)^k x^{4k+2}}{(2k+1)!}$, valid for all x .

D) $\sum_{k=0}^{\infty} x^k$, valid for $|x| < 1$

E) $x^2 + \frac{x^4}{2} + \frac{x^4}{6} + \dots$, valid for $|x| < 2$.

15. A bridge hand consists of 13 cards chosen at random from a deck of 52. You are dealt a hand. What is the probability that it has at least one spade? *Hint: 39 of the cards in the deck are NOT spades.*

- A) $1 - \binom{39}{13} / \binom{52}{13}$ B) $\binom{39}{13} / \binom{52}{13}$ C) $\binom{39}{13}$ D) $\binom{52}{13}$ E) $\binom{13}{13}$

16. Find the Taylor polynomial of degree 4 for the function $f(x) = \frac{1}{1-x}$.

- A) $1 - x + x^2 - x^3 + x^4$ B) $1 + x + x^2 + x^3 + x^4$ C) $1 + x + x^2 + x^3$
D) $1 + \frac{x}{2} + \frac{x^3}{3} + \frac{x^4}{4}$ E) $\sin x$

17. Suppose an initial amount of \$500 is invested at an annual rate of 6.5% compounded continuously. How much will there be at the end of two years?

- A) $500\left(1 + \frac{0.065}{12}\right)^{(20)12}$ B) $500e^{(20)65}$ C) $500e^{(20)0.065}$ D) 501 E) 1000

18. Calculate $\binom{15}{2}$.

A) 30

B) 96

C) 210

D) -7

E) 105

19. Use the linear approximation and estimate the size of the error. $f(x) = \sin x$ where $x = 0.02$

- A) $1.02, |error| \leq \frac{(0.02)^2}{2}$ B) $0.02, |error| \leq \frac{(0.02)^2}{2}$ C) $-0.02, |error| \leq \frac{(0.02)^2}{2}$
D) $0.02, |error| \leq \frac{(0.02)^3}{2000}$ E) None of the above.

20. Find the power series representation of the following function and determine the interval on which it is valid: $t(x) = xe^x$.

A) $\sum_{k=0}^{\infty} \frac{x^{k+1}}{k!}$, valid for all x

B) $\sum_{k=0}^{\infty} \frac{x^k}{k!}$, valid for all x .

C) $x + x^2 + \frac{x^3}{2!}$, valid for all x .

D) $\sum_{k=0}^{\infty} \frac{(-1)^k x^{k+1}}{k!}$, valid for all x .

E) None of the above.