1. Suppose f is a continuous function whose average value on the interval [0,3] is 1, and suppose that $\int_0^1 f(x)\,dx = 2$.

Then $\int_1^3 f(x)\,dx =$

(A) 2 (B) 3 (C) -1 (D) 0 (E) 1

2. The area of the region shown in the diagram is equal to

(A) $\pi - \frac{1}{3}$ (B) $\frac{\pi}{4} - \frac{2}{3}$ (C) $\pi + \frac{1}{3}$ (D) $\frac{\pi}{4} - \frac{1}{3}$ (E) $\frac{\pi}{2} + \frac{1}{3}$

3. The total area between the curve $y = x^3 + x^2 - 2x$ $(-2 \le x \le 1)$ and the x-axis is

(A) $\frac{5}{4}$ (B) $\frac{37}{12}$ (C) 3 (D) $\frac{9}{4}$ (E) 13

4. $$\int_0^{\pi/2} \tan^7\left(\frac{x}{2}\right) \sec^2\left(\frac{x}{2}\right) dx =$$

(A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) $\frac{1}{8}$ (E) $\frac{1}{16}$

5. $$\int (2x^2 - 1)^{99} x \, dx =$$

(A) $\frac{(2x^2 - 1)^{100}}{25} + C$ (B) $\frac{(2x^2 - 1)^{100}}{50} + C$

(C) $\frac{(2x^2 - 1)^{100}}{100} + C$ (D) $\frac{(2x^2 - 1)^{100}}{200} + C$

(E) $\frac{(2x^2 - 1)^{100}}{400} + C$

6. An application of the trapezoidal rule with $n = 3$ steps shows that

$$\int_{-1}^{2} \sqrt{1 + 4x^2 - x^4}\, dx$$

is approximately equal to

(A) $\frac{17}{4}$ (B) $\frac{9}{2}$ (C) $\frac{\sqrt{15}}{2}$ (D) $\sqrt{\frac{15}{2}}$ (E) $\frac{20}{3}$

7. The area enclosed by the curves $y = 2x + \sqrt{x}$ and $y = x + 3\sqrt{x}$ is

(A) $\frac{8}{3}$ (B) $2 - \frac{8\sqrt{2}}{3}$ (C) $\frac{5}{6}$ (D) $\frac{5}{3}$ (E) $\frac{4}{3}$

8. The area enclosed between the two parabolas

$$x = 4 - y^2 ,$$

$$x = y^2 + 2y ,$$

is equal to

(A) $\frac{9}{2}$ (B) $\frac{5}{2}$ (C) 3 (D) 9 (E) 4

9. The region shown in the diagram is revolved about the x-axis. The volume of the solid generated in this way is

(A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) $\frac{\pi}{2}$ (D) π (E) $\frac{\pi}{4}$

10. The same region as in problem 9 is revolved about the line $y = -1$. The volume of the resulting solid is

(A) $\int_0^{\pi/4} \pi (\sec^2 x - 1)\, dx$

(B) $\int_0^{\pi/4} \pi (2 \sec x + \sec^2 x)\, dx$

(C) $\int_0^{\pi/4} \pi (1 + \sec^2 x)\, dx$

(D) $\int_0^{\pi/4} \pi \sec^2 x\, dx$

(E) $\int_0^{\pi/4} \pi (1 + \sec x)^2\, dx$

11. The region shown in the diagram is revolved about the x-axis. The volume of the solid generated in this way is

(A) 16π (B) $\frac{15\pi}{2}$ (C) 6π (D) 8π (E) $\frac{8\pi}{3}$

12. The same region as in problem 11 above is revolved about the line $x = 4$. The volume of the resulting solid is

(A) $\int_0^4 \pi\left(4 - \left(2 - \sqrt{4 - x}\right)^2\right) dx$

(B) $\int_0^4 2\pi\,(4 - x)\left(2 - \sqrt{4 - x}\right) dx$

(C) $\int_0^4 2\pi\, x\left(2 - \sqrt{4 - x}\right) dx$

(D) $\int_0^4 \pi\left(4 - \sqrt{4 - x}\right)^2 dx$

(E) $\int_0^4 2\pi\,(4 - x)\left(2 - \sqrt{4 - x}\right)^2 dx$

13. A certain solid has a base which is the region between the curve $y = 2\sqrt{\sin x}$ and the interval $[0, \pi]$ on the x-axis. The cross sections perpendicular to the x-axis are vertical equilateral triangles with bases running from the x-axis to the curve. The volume of the solid is

(A) $\frac{\sqrt{3}}{2}$ (B) 4 (C) 1 (D) $2\sqrt{3}$ (E) $\frac{8}{3}$

14. The y–coordinate of the center of mass of a thin plate of constant density covering the region shown in the diagram is

(A) $\frac{3}{8}$ (B) $\frac{1}{6}$ (C) $\frac{1}{2}$ (D) $\frac{1}{3}$ (E)

$\frac{2}{3}$

15. The length of the curve $y = \frac{1}{3}(x^2 + 2)^{3/2}$ from $x = 0$ to $x = 3$ is

(A) $\frac{22}{3}$ (B) 11 (C) 12 (D) $11\sqrt{11}$ (E) $\frac{121}{9}$

16. If the curve of problem 15 is revolved about the x-axis, the area of the resulting surface is

(A) $\int_0^3 2\pi x^2 (x^2 + 1)\, dx$

(B) $\int_0^3 2\pi x(x^2 + 2)^{3/2}\, dx$

(C) $\int_0^3 \frac{\pi}{9} (x^2 + 2)^3\, dx$

(D) $\int_0^3 2\pi x(x^2 + 1)\, dx$

(E) $\int_0^3 \frac{2\pi}{3} (x^2 + 1)(x^2 + 2)^{3/2}\, dx$

17. An elevator with a motor at the top has a cable weighing 5 lb/ft. When the car is at the first floor, 200 ft of cable are paid out, and effectively 0 ft are out when the car is at the top floor. When the motor takes the car from the first floor to the top, the amount of work it does just lifting the cable is (in ft–lb)

(A) 200,000 (B) 80,000 (C) 160,000

(D) 100,000 (E) 120,000