1. $\lim\limits_{x \varnothing 1} \dfrac{\tan(x-1)}{x^2 + x - 2} =$

(A) ∞ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 1 (E) $\frac{3}{2}$

2. The tangent line to the graph of the equation

$$y = (x^3 - \pi x^2 + 1) \tan\left(\frac{x}{4}\right)$$

at the point $(\pi, 1)$ has slope equal to

(A) $\pi^2 + 2$ (B) $2\pi^2$ (C) π^2 (D) $\pi^2 + \frac{1}{2}$ (E) $\frac{1}{2}\pi^2$

3. A certain function f satisfies

$$f(2) = 2, \; f'(2) = 3.$$

If $g(x) = \dfrac{x}{f(x)}$, then $g'(2) =$

(A) $\frac{1}{3}$ (B) $-\frac{1}{2}$ (C) -1 (D) 0 (E) 1

4. Just one of these statements is true.

(A) If $f(x)$ is discontinuous at $x = a$, then $\lim_{x \to a} f(x)$ does not exist

(B) If $\lim_{x \to a} f(x)$ exists, then $f(x)$ is differentiable at $x = a$

(C) If $f(x)$ is differentiable at $x = a$, then $f(x)$ is continuous at $x = a$

(D) If $\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x)$, then $f(x)$ is continuous at $x = a$

(E) If $f(x)$ is continuous at $x = a$, then $f(x)$ is differentiable at $x = a$

5. The tangent line to the graph of the equation $xy^2 + x + y + 1 = 0$ at the point $(-1, 1)$ has equation

(A) $y = 3x + 4$ (B) $y = x + 2$ (C) $y = \frac{2}{3}x + \frac{5}{3}$

(D) $y = \frac{1}{2}x + \frac{3}{2}$ (E) $y = 2x + 3$

6. The position s of a body moving on a straight line is given by the equation

$$s = \int_{1}^{t^2} \sqrt{(x-2)^3 + 1}\; dx \qquad (t \geq 1).$$

At time $t = 2$, the velocity of the body is

(A) 3 (B) 2 (C) 1 (D) 12 (E) 6

7. The approximate value of

$$f(x) = \frac{x^3 - x}{x^2 + 3} + 1 ,$$

when $x = 0.998$, found by the method of linearization or differentials, is

(A) 0.995 (B) 0.996 (C) 0.997 (D) 0.998 (E) 0.999

8. A spherical hailstone is growing as it falls, by accumulating ice at a rate of 2 cubic centimeters per minute. At a certain instant its radius is 1 centimeter, and at that time the rate (in centimeters per minute) at which the radius is increasing is

(A) $\frac{\pi}{2}$ (B) $\frac{1}{6\pi}$ (C) $\frac{1}{2\pi}$ (D) $\frac{1}{4\pi}$ (E) $\frac{\pi}{4}$

9. The local extreme values of the function

$$y = \frac{x^3 - 3x^2}{3x - 1}$$

are given by

(A) a local maximum at $x = 0$ and a local minimum at $x = 1$

(B) a local maximum at $x = \frac{1}{3}$ only

(C) a local minimum at $x = 0$, and a local maximum at $x = 1$

(D) a local minimum at $x = 0$ only

(E) a local maximum at $x = 1$ only

10. The graph of $y = 2x^6 - 5x^4$ has

(A) two inflection points, at $x = -1$ and $x = 1$

(B) three inflection points, at $x = -1$, $x = 0$ and $x = 1$

(C) only one inflection point, at $x = 0$

(D) only one inflection point, at $x = -1$

(E) only one inflection point, at $x = 1$

11. The graph of $y = \frac{x^2 - 2x + 1}{x^2 + x - 2}$ has asymptotes

(A) $x = 1$ and $x = -2$

(B) $x = -1$, $x = -2$ and $y = 1$

(C) $x = -2$ and $y = 0$

(D) $x = 1$, $x = -2$ and $y = 1$

(E) $x = -2$ and $y = 1$

12. The U.S. Postal Service will accept a box for domestic shipment only if the sum of the length and girth (distance around) does not exceed 108 inches. The length (in inches) of the largest acceptable box with a square end is

(A) 72 (B) $18\sqrt{2}$ (C) 54 (D) 36 (E) 18

13. A company estimates that the marginal cost (in dollars per item) when producing x items is $2 + \frac{100}{x^2} - 0.02x$. If the cost of producing 10 items is \$110, then the cost of producing 100 items is

(A) \$290 (B) \$131 (C) \$1100 (D) \$352 (E) \$200

14. An object moves along a straight line with acceleration given by $a = \pi \sin\left(\frac{t}{2}\right) - 2$. At time $t = 0$, it is at the position $s = 0$ and has velocity $v = 3$. At time $t = \pi$, it is at the position $s =$

(A) $-\pi^2 + 2\pi$ (B) $\pi^2 - \pi$ (C) $\pi^2 + 2\pi$
(D) $-\pi^2 - \pi$ (E) $\pi^2 + 7\pi$

15. The area under the graph of
$y = \cos^2 x$, $0 \le x \le \frac{\pi}{2}$, is

(A) 2 (B) π (C) $\frac{1}{4}$ (D) $\frac{\pi}{2}$ (E) $\frac{\pi}{4}$

16. $\int x \sec^2(x^2)\ dx =$

(A) $x \tan(x^2) + \frac{1}{2} x^2 \sec^2(x^2) + C$

(B) $\frac{1}{2} x^3 \tan(x^2) + C$

(C) $2x^2 \sec^2(x^2) + C$

(D) $\frac{1}{2} \tan(x^2) + C$

(E) $\frac{1}{4} x^2 \tan(x^2) + C$

17. $\int_0^2 \frac{x^2}{\sqrt{1 + x^3}}\ dx =$

(A) 2 (B) $\frac{1}{3}$ (C) 12 (D) $\frac{2}{9}$ (E) $\frac{4}{3}$

18. The area of the region enclosed by the parabolas $y = x^2$ and $x = y^2$ is

(A) $\frac{1}{4}$ (B) $\frac{2}{3}$ (C) $\frac{2}{5}$

(D) $\frac{1}{3}$ (E) $\frac{1}{2}$

19.

A solid of revolution is formed by revolving the region under the graph of a function $y = f(x)$, $1 \le x \le 4$, about the x-axis. The values of $f(1)$, $f(2)$, $f(3)$, $f(4)$ are as shown. Application of the trapezoidal rule to a certain integral shows that the volume of the solid is approximately

(A) 14π (B) 10π (C) 6π (D) 18π (E) 8π

20. The region shown in the diagram is revolved about the x-axis. The volume of the solid generated in this way is

(A) $\frac{15\pi}{2}$ (B) 6π

(C) 8π (D) $\frac{8\pi}{3}$ (E) 16π

21. The region shown in the diagram is revolved about the line $x = 1$.

The volume of the solid generated in this way is

(A) $\frac{8\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{6}$ (E) $\frac{5\pi}{6}$

22. The curve in the diagram represents the solution to the initial value problem $\frac{dy}{dx} = \sqrt{x^2 + 2x}$, $y(0) = 0$.

The length of the curve from $x = 0$ to $x = 1$ is

(A) $\frac{3}{2}$ (B) $\sqrt{3}$ (C) 2 (D) $\frac{4}{3}$ (E) $\frac{\sqrt{3} + 1}{2}$

23. The curve $y = 2x^{3/2}$, $0 \le x \le 1$, is revolved about the y-axis. The area of the surface generated in this way is given by

(A) $\int_0^1 2\pi x \sqrt{1 + 3x^{1/2}}\ dx$

(B) $\int_0^1 4\pi x^{3/2} \sqrt{1 + 4x^3}\ dx$

(C) $\int_0^1 2\pi x\sqrt{1 + 4x^3}\ dx$

(D) $\int_0^1 4\pi x^{3/2} \sqrt{1 + 9x}\ dx$

(E) $\int_0^1 2\pi x\sqrt{1 + 9x}\ dx$

24. The x-coordinate of the center of mass of a thin plate of constant density covering the region shown in the diagram is

(A) $\frac{3}{5}$ (B) $\frac{2}{5}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$ (E) $\frac{1}{3}$

25. A bungee cord whose natural length is 50m is stretched to a length of 52m by a force of 12N. Assume Hooke's Law applies. The work required to stretch the cord from a length of 52m to a length of 55m is (in units of Nm)

(A) 21 (B) 48 (C) 36 (D) 63 (E) 24