1. The average value of an integrable function f on the interval [a,b] is defined as

$$\frac{1}{b-a}\int_a^b f(x)\,dx.$$

Using this definition, compute the average value of

$$f(x) = \frac{\sec^2 x}{(1 + 7\tan x)^{2/3}}$$

on the interval $\left[0, \frac{\pi}{4}\right]$.

(A) $\frac{17}{10\pi}$ (B) $\frac{5}{3\pi}$ (C) $\frac{13}{8\pi}$ (D) $\frac{7}{4\pi}$ (E) $\frac{12}{7\pi}$

2. Suppose that f has a <u>negative</u> derivative for all values of x and that $f(1) = 0$. How many of the following statements must be true of the function

$$h(x) = \int_0^x f(t)\,dt\ ?$$

i) h is a twice-differentiable function of x.

ii) The graph of h has a horizontal tangent at $x = 1$.

iii) h has a local maximum at $x = 1$.

iv) The graph of h has an inflection point at $x = 1$.

(A) 4 (B) 3 (C) 2 (D) 1 (E) 0

3. Some values of a function f are given in the table below

x	0	3	6	9	12
f(x)	5	9	7	11	10

By Simpson's Rule, $\int_0^{12} f(x)dx$ is approximately

(A) 104 (B) 109 (C) 98 (D) 126 (E) 112

4. Find the area of the region between the graphs of $y = x^2 - x - 4$ and $y = x - 1$.

(A) 10
(B) $\frac{41}{4}$
(C) $\frac{32}{3}$
(D) $\frac{65}{6}$
(E) 11

5. Find the area of the "triangular" region bounded on the left by $y = \sqrt{x}$, on the right by $y = 6 - x$ and below by $y = 1$.

(A) $\frac{7}{3}$

(B) $\frac{5}{2}$

(C) $\frac{9}{4}$

(D) $\frac{17}{8}$

(E) $\frac{13}{6}$

6. A solid lies between planes perpendicular to the x-axis at $x = \frac{\pi}{4}$ and $x = \frac{5}{4}\pi$. The cross sections between these planes are circular discs whose diameters run from the curve $y = 2\cos x$ to the curve $y = 2\sin x$. The volume of this solid is

(A) $2\pi^{3/2}$ (B) $2\sqrt{2}\,\pi$ (C) 4π (D) π^2 (E) $2\sqrt{3}\,\pi$

7. Let R be the region bounded by the semi-circle $y = \sqrt{25 - x^2}$ and the line $y = 4$. Find the volume of the solid obtained by revolving R about the x-axis.

(A) 36π (B) 25π (C) 30π (D) 32π (E) 24π

8. The volume of the solid generated by revolving the region bounded by the parabola $y = x^2$ and the line $y = 1$ about the line $y = 2$ is

(A) $\int_{-1}^{1} \pi\left(1 - x^2\right)^2 dx$

(B) $\int_{-1}^{1} \pi\left[\left(2 - x^2\right)^2 - 1\right] dx$

(C) $\int_{0}^{1} 2\pi\,(2 - y)(1 - y)\, dy$

(D) $\int_{-1}^{1} \pi\left[2^2 - \left(2 - x^2\right)^2\right] dx$

(E) $\int_{0}^{1} 2\pi\, y\,(2 - y)\, dy$

9. The volume of the solid generated by revolving the region bounded by the parabolas $x = 3y^2 - 2$ and $x = y^2$ about the x–axis is

(A) 2π

(B) $\frac{2}{3}\pi$

(C) $\frac{4}{3}\pi$

(D) π

(E) $\frac{5}{2}\pi$

10. The length of the curve $y = \left(x^2 - \frac{2}{3}\right)^{3/2}$ from $x = 1$ to $x = 2$ is

(A) 6 (B) $4\sqrt{2}$ (C) $\frac{27}{4}$ (D) $3^{3/2}$ (E) $\frac{16}{3}$