1. Which of the following statements about the function $y = f(x)$ graphed here are true, and which are false?

Which one of the following is true?

(A) Neither $\lim_{x \varnothing 1} f(x)$ nor $\lim_{x \varnothing 2} f(x)$ exists

(B) $\lim_{x \varnothing H} f(x) = 1$ and $\lim_{x \varnothing 2} f(x) = 2$

(C) $\lim_{x \varnothing 1^-} f(x)$ does not exist but $\lim_{x \varnothing 0} f(x)$ does

(D) Both $\lim_{x \varnothing 1^-} f(x)$ and $\lim_{x \varnothing 2} f(x)$ exist

(E) $\lim_{x \varnothing 0^-} f(x) \neq \lim_{x \varnothing 0^+} f(x)$

2. $\lim_{y \varnothing 0} \dfrac{5y^3 + 8y^2}{3y^4 - 16y^2} = ?$

(A) $\frac{5}{3}$ (B) $\frac{8}{3}$ (C) $-\frac{1}{2}$ (D) $-\frac{5}{16}$ (E) 0

3. $\lim_{x \varnothing 0} 6x^2 (\cot x)(\csc 2x) = ?$

(A) 3 (B) $\frac{1}{4}$ (C) 0 (D) 12 (E) doesn't exist

4. If, for $x \geq 0$,

$$3x^2 - 5x - 1 \leq (x^2 - 7x + 1)\, f(x) \leq 3x^2 - x + 4,$$

Then $\lim_{x \to \infty} f(x) = ?$

(A) 2 (B) ∞ (C) -5 (D) 0 (E) 3

5. For $x \neq 2$, $f(x) = \dfrac{x^2 + 6x - 16}{x - 2}$. If f is also defined and continuous at $x = 2$, then $f(2) = ?$

(A) 1 (B) 10 (C) $-\frac{1}{2}$ (D) 8 (E) -3

6. At how many points on the graph of $y = x - \dfrac{1}{x}$ is the tangent line parallel to the line $2x - y = 5$?

(A) 0 (B) 4 (C) 2 (D) 3 (E) 1

7. If $f(x) = 2x\sqrt{1 + 3x}$, then $f'(1) = ?$

(A) 5 (B) $\frac{19}{3}$ (C) $2\sqrt{3}$ (D) $\frac{11}{2}$ (E) 8

8. The slope of the curve $x^2 + \frac{x}{y} = 6$ at the point $(3, -1)$ is

(A) 3(B) $\frac{16}{5}$ (C) $\frac{7}{9}$ (D) $-\frac{4}{15}$ (E) $\frac{5}{3}$

9. If $f(x) = \left[x^3 + (2x - 1)^3 \right]^3$, then $f'(1) = ?$

(A) 108 (B) 72 (C) 54 (D) 96 (E) 48

10. If $f(x) = \dfrac{\tan x - 1}{\sec x}$, then $f'\left(\frac{\pi}{4}\right) = ?$

(A) 0 (B) $\sqrt{2}$ (C) $\frac{1}{2}$ (D) $\sqrt{3}$ (E) 1

11. If $y = \left(x^4 - 3x^2 + 1\right)^{10}$, use the differential of y to approximate the change in y when x changes from 1 to 1.01.

(A) 0.4 (B) 0.1 (C) 0.3 (D) 0.2 (E) 0.5

12. The local extreme values of the function $y = \dfrac{x^5}{(x-2)^3}$ are given by

(A) a local maximum at $x = 3$ only

(B) a local minimum at $x = 5$ and a local maximum at $x = 0$

(C) a local minimum at $x = 5$ only

(D) a local maximum at $x = 0$ only

(E) a local maximum at $x = 5$ and a local minimum at $x = 0$

13. The graph of $y = \dfrac{9x^2 + 3x - 2}{3x^2 + 2x - 1}$ has asymptotes

(A) $x = -1$, $x = \frac{1}{3}$ and $y = 3$

(B) $x = -1$ and $y = 0$

(C) $x = \frac{1}{3}$ and $y = 3$

(D) $x = -1$ and $x = \frac{1}{3}$

(E) $x = -1$ and $y = 3$

14. A projectile fired upward from the surface of the moon is to reach a maximum height of 1000 ft. What must its initial velocity (in ft/sec) be? The acceleration due to lunar gravity is 5ft/sec^2.

(A) 100 (B) 80 (C) 150 (D) 110 (E) 120

15. A rubbish heap in the shape of a cube is being compacted. If the volume decreases at the rate of 2 cubic meters per minute, the rate of change of surface area of the cube when the volume is 27 cubic meters, in square meters per minute, is

(A) -2 (B) $-\frac{5}{3}$ (C) -3 (D) $-\frac{8}{3}$ (E) $-\frac{10}{3}$

16. A house at A is in the woods 12 miles north of an east-west road, the nearest point of which is B. At C, 5 miles east of B on the road, is an electric power substation. If the power line is built to join C to A, it costs 3 times as much per mile through the woods as along the highway. The line will either go

straight from C to A or along the road from C to a point P part way toward B and then through the woods to A. The cheapest plan is to go

(A) $\sqrt{2}$ miles west to P

(B) straight to A

(C) $5 - 3\sqrt{2}$ miles west to P

(D) 3 miles west to P

(E) $4 - \sqrt{3}$ miles west to P

17. The graph of $y = 2x^6 - 5x^4 + x + 1$

(A) has only one point of inflection

(B) is concave downwards on $(-1, 1)$

(C) is concave upwards on $(-1, 0)$ and $(1,\infty)$

(D) has three points of inflection

(E) is concave downwards on $(-\infty,-1)$ and $(1,\infty)$

18. If $\frac{dy}{dx} = \frac{4x}{(x^2 - 3)^2}$ and $y(1) = 3$

then $y(2) = ?$

(A) 1 (B) $\frac{2}{3}$ (C) -1 (D) $\frac{5}{2}$ (E) 0

19. Consider the function $g(x) = \cos x,\ \frac{\pi}{6} \leq x \leq \frac{\pi}{2}$.

Let $P = \left\{ \frac{\pi}{6} = x_0, x_1, x_2, \ldots, x_n = \frac{\pi}{2} \right\}$ be a typical partition of $\left[\frac{\pi}{6}, \frac{\pi}{2} \right]$ and let $x_{k-1} \leq c_k \leq x_k$ for $k = 1, \ldots, n$. Then

$$\lim_{\|P\| \to 0} \left(\sum_{k=1}^{n} g(c_k)\ \Delta x_k \right) =$$

(A) $\frac{\pi}{3}$ (B) $\frac{1}{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) 1 (E) $\frac{\sqrt{3}}{3}$

20. The area between the region bounded by the graphs of

$$y = x^2 \text{ and } y = x$$

is

(A) $\frac{1}{6}$ (B) $\frac{5}{24}$ (C) $\frac{1}{12}$ (D) $\frac{1}{4}$ (E) $\frac{7}{36}$

21. The volume of the solid generated by revolving about the x–axis the region in the 1st quadrant bounded by the graphs of $y = x$, $y = \frac{1}{x}$, $x = 1$ and $x = 2$ is

(A) 2π (B) $\frac{7}{4}\pi$ (C) $\frac{11}{6}\pi$ (D) $\frac{5}{3}\pi$ (E) $\frac{13}{6}\pi$

22.

A solid of revolution is formed by rotating the region under the graph of the function $y = f(x)$ $1 \le x \le 4$, about the y–axis. The values of $f(1)$, $f(2)$, $f(3)$, $f(4)$ are as shown. Application of the trapezoidal rule to a certain integral shows that the volume of the solid is approximately

(A) 35π (B) 28π (C) 32π (D) 34π (E) 30π

23. The curve $y = x^2$, $0 \le x \le 3$ is revolved about the x–axis. The area of the surface generated in this way is given by

(A) $\int_0^3 2\pi x^2 \sqrt{1 + 4x^2}\, dx$

(B) $\int_0^3 2\pi x \sqrt{1 + 4x^2}\, dx$

(C) $\int_0^3 2\pi x^2 \sqrt{1 + x^4}\, dx$

(D) $\int_0^3 2\pi x^2 \sqrt{1 + 2x}\, dx$

(E) $\int_0^3 2\pi x \sqrt{1 + 2x}\, dx$

24. The y-coordinate of the center of mass of a thin plate of constant (uniform) density covering the region shown in the diagram is

(A) $\frac{1}{3}$

(B) $\frac{\pi}{8}$

(C) $2\sqrt{3}$

(D) $\frac{\pi}{6}$

(E) $\frac{9}{4}\sqrt{2}$

25. A swimming pool has the shape of a right cirvular cylinder with radius 10 ft. and depth 8 ft. If the pool contains water (weighing 62.5 lb/ft^3) to a depth of 5 ft. find the work (in ft-lbs) required to pump all but 1 ft. of water to the top of the pool

(A) $171875\,\pi$

(B) $10000\,\pi$

(C) $156250\,\pi$

(D) $187500\,\pi$

(E) $125000\,\pi$