1. Which of the following statements about the function \( y = f(x) \) graphed here are true, and which are false?

   Which one of the following is true?

   (A) Neither \( \lim_{x \to 1} f(x) \) nor \( \lim_{x \to 2} f(x) \) exists
   (B) \( \lim_{x \to 1} f(x) = 1 \) and \( \lim_{x \to 2} f(x) = 2 \)
   (C) \( \lim_{x \to 1^-} f(x) \) does not exist but \( \lim_{x \to 0} f(x) \) does
   (D) Both \( \lim_{x \to 1^-} f(x) \) and \( \lim_{x \to 2} f(x) \) exist
   (E) \( \lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x) \)

2. \( \lim_{y \to 0} \frac{5y^3 + 8y^2}{3y^4 - 16y^2} = ? \)

   (A) \( \frac{5}{3} \)  
   (B) \( \frac{8}{3} \)  
   (C) \( -\frac{1}{2} \)  
   (D) \( -\frac{5}{16} \)  
   (E) 0

3. \( \lim_{x \to 0} 6x^2 \cot x \csc 2x = ? \)
4. If, for \( x \geq 0, \)

\[
3x^2 - 5x - 1 \leq (x^2 - 7x + 1) f(x) \leq 3x^2 - x + 4,
\]

Then \( \lim_{x \to \infty} f(x) = ? \)

(A) 2  (B) \( \infty \)  (C) -5  (D) 0  (E) 3

5. For \( x \neq 2, f(x) = \frac{x^2 + 6x - 16}{x - 2} \). If \( f \) is also defined and continuous at \( x = 2, \)
then \( f(2) = ? \)

(A) 1  (B) 10  (C) \(-\frac{1}{2}\)  (D) 8  (E) -3

6. At how many points on the graph of \( y = x - \frac{1}{x} \) is the tangent line parallel to the line \( 2x - y = 5? \)
7. If \( f(x) = 2x\sqrt{1 + 3x} \), then \( f'(1) = ? \)

(A) 5          (B) \( \frac{19}{3} \)          (C) \( 2\sqrt{3} \)          (D) \( \frac{11}{2} \)          (E) 8

8. The slope of the curve \( x^2 + \frac{x}{y} = 6 \) at the point \( (3, -1) \) is

(A) 3          (B) \( \frac{16}{5} \)          (C) \( \frac{7}{9} \)          (D) \( -\frac{4}{15} \)          (E) \( \frac{5}{3} \)

9. If \( f(x) = \left[ x^3 + (2x - 1)^3 \right]^3 \), then \( f'(1) = ? \)

(A) 108          (B) 72          (C) 54          (D) 96          (E) 48
10. If \( f(x) = \frac{\tan x - 1}{\sec x} \), then \( f'(\frac{\pi}{4}) = ? \)

(A) 0  
(B) \( \sqrt{2} \)  
(C) \( \frac{1}{2} \)  
(D) \( \sqrt{3} \)  
(E) 1

11. If \( y = (x^4 - 3x^2 + 1)^{10} \), use the differential of \( y \) to approximate the change in \( y \) when \( x \) changes from 1 to 1.01.

(A) 0.4  
(B) 0.1  
(C) 0.3  
(D) 0.2  
(E) 0.5

12. The local extreme values of the function \( y = \frac{x^5}{(x - 2)^3} \) are given by

(A) a local maximum at \( x = 3 \) only
(B) a local minimum at \( x = 5 \) and a local maximum at \( x = 0 \)
(C) a local minimum at \( x = 5 \) only
(D) a local maximum at \( x = 0 \) only
(E) a local maximum at \( x = 5 \) and a local minimum at \( x = 0 \)

13. The graph of \( y = \frac{9x^2 + 3x - 2}{3x^2 + 2x - 1} \) has asymptotes

(A) \( x = -1, \ x = \frac{1}{3} \) and \( y = 3 \)
14. A projectile fired upward from the surface of the moon is to reach a maximum height of 1000 ft. What must its initial velocity (in ft/sec) be? The acceleration due to lunar gravity is 5 ft/sec².

(A) 100  (B) 80  (C) 150  (D) 110  (E) 120

15. A rubbish heap in the shape of a cube is being compacted. If the volume decreases at the rate of 2 cubic meters per minute, the rate of change of surface area of the cube when the volume is 27 cubic meters, in square meters per minute, is

(A) −2  (B) −\frac{5}{3}  (C) −3  (D) −\frac{8}{3}  (E) −\frac{10}{3}

16. A house at A is in the woods 12 miles north of an east-west road, the nearest point of which is B. At C, 5 miles east of B on the road, is an electric power substation. If the power line is built to join C to A, it costs 3 times as much per mile through the woods as along the highway. The line will either go
straight from C to A or along the road from C to a point P part way toward B and then through the woods to A. The cheapest plan is to go

(A) $\sqrt{2}$ miles west to P
(B) straight to A
(C) $5 - 3\sqrt{2}$ miles west to P
(D) 3 miles west to P
(E) $4 - \sqrt{3}$ miles west to P

17. The graph of $y = 2x^6 - 5x^4 + x + 1$

(A) has only one point of inflection
(B) is concave downwards on $(-1, 1)$
(C) is concave upwards on $(-1, 0)$ and $(1, \infty)$
(D) has three points of inflection
(E) is concave downwards on $(-\infty, -1)$ and $(1, \infty)$

18. If \( \frac{dy}{dx} = \frac{4x}{(x^2 - 3)^2} \) and \( y(1) = 3 \)

then \( y(2) = ? \)
19. Consider the function \( g(x) = \cos x, \quad \frac{\pi}{6} \leq x \leq \frac{\pi}{2} \).

Let \( P = \left\{ \frac{\pi}{6} = x_0, x_1, x_2, \ldots, x_n = \frac{\pi}{2} \right\} \) be a typical partition of \( \left[ \frac{\pi}{6}, \frac{\pi}{2} \right] \) and let \( x_{k-1} \leq c_k \leq x_k \) for \( k = 1, \ldots, n \). Then

\[
\lim_{|P| \to 0} \left( \sum_{k=1}^{n} g(c_k) \Delta x_k \right) =
\]

(A) \( \frac{\pi}{3} \) \hspace{1cm} (B) \( \frac{1}{2} \) \hspace{1cm} (C) \( \frac{1}{\sqrt{2}} \) \hspace{1cm} (D) 1 \hspace{1cm} (E) \( \frac{\sqrt{3}}{3} \)

20. The area between the region bounded by the graphs of \( y = x^2 \) and \( y = x \)
is

\[
\begin{align*}
(A) \quad & \frac{1}{6} \\
&B \quad \frac{5}{24} \\
(C) \quad & \frac{1}{12} \\
(D) \quad & \frac{1}{4} \\
&E \quad \frac{11}{6} \\
\end{align*}
\]

21. The volume of the solid generated by revolving about the x–axis the region in the 1\textsuperscript{st} quadrant bounded by the graphs of \( y = x \), \( y = \frac{1}{x} \), \( x = 1 \) and \( x = 2 \) is

\[
\begin{align*}
(A) \quad & 2\pi \\
(B) \quad & \frac{7}{4} \pi \\
(C) \quad & \frac{11}{6} \pi \\
(D) \quad & \frac{5}{3} \pi \\
(E) \quad & \frac{13}{6} \pi \\
\end{align*}
\]
22. A solid of revolution is formed by rotating the region under the graph of the function $y = f(x)$, $1 \leq x \leq 4$, about the y-axis. The values of $f(1)$, $f(2)$, $f(3)$, $f(4)$ are as shown. Application of the trapezoidal rule to a certain integral shows that the volume of the solid is approximately

(A) $35\pi$  (B) $28\pi$  (C) $32\pi$  (D) $34\pi$  (E) $30\pi$

23. The curve $y = x^2$, $0 \leq x \leq 3$ is revolved about the x-axis. The area of the surface generated in this way is given by

(A) $\int_{0}^{3} 2\pi x^2 \sqrt{1 + 4x^2} \, dx$

(B) $\int_{0}^{3} 2\pi x \sqrt{1 + 4x^2} \, dx$

(C) $\int_{0}^{3} 2\pi x^2 \sqrt{1 + x^4} \, dx$

(D) $\int_{0}^{3} 2\pi x^2 \sqrt{1 + 2x} \, dx$

(E) $\int_{0}^{3} 2\pi x \sqrt{1 + 2x} \, dx$
24. The y-coordinate of the center of mass of a thin plate of constant (uniform) density covering the region shown in the diagram is

(A) \( \frac{1}{3} \)
(B) \( \frac{\pi}{8} \)
(C) \( 2\sqrt{3} \)
(D) \( \frac{\pi}{6} \)
(E) \( \frac{9}{4}\sqrt{2} \)

25. A swimming pool has the shape of a right circular cylinder with radius 10 ft. and depth 8 ft. If the pool contains water (weighing 62.5 lb/ft\(^3\)) to a depth of 5 ft. find the work (in ft-lbs) required to pump all but 1 ft. of water to the top of the pool

(A) \( 171875 \pi \)
(B) \( 10000 \pi \)
(C) \( 156250 \pi \)
(D) \( 187500 \pi \)
(E) \( 125000 \pi \)