Exam I
September 25, 2001

11. By definition $f'(1) = \lim_{h\to 0} \dfrac{\sqrt{1+h}-\sqrt{1}}{h} = \lim_{h\to 0} \dfrac{\sqrt{1+h}-\sqrt{1}}{h} \cdot \dfrac{\sqrt{1+h}+\sqrt{1}}{\sqrt{1+h}+\sqrt{1}}$
$= \lim_{h\to 0} \dfrac{\sqrt{1+h}^2-\sqrt{1}^2}{h} \cdot \dfrac{1}{\sqrt{1+h}+\sqrt{1}} = \lim_{h\to 0} \dfrac{1+h-1}{h} \cdot \dfrac{1}{\sqrt{1+h}+\sqrt{1}} \lim_{h\to 0} \dfrac{h}{h} \cdot \dfrac{1}{\sqrt{1+h}+\sqrt{1}}$
$= \lim_{h\to 0} \dfrac{1}{\sqrt{1+h}+\sqrt{1}} = \dfrac{1}{\sqrt{1}+\sqrt{1}} = \dfrac{1}{2}$, where the next to the last equality follows since $\sqrt{x+1}$ is continuous at 0.

OR

By definition $f'(1) = \lim_{x\to 1} \dfrac{\sqrt{x}-\sqrt{1}}{x-1} = \lim_{x\to 1} \dfrac{\sqrt{x}-\sqrt{1}}{x-1} \cdot \dfrac{\sqrt{x}+\sqrt{1}}{\sqrt{x}+\sqrt{1}}$
$= \lim_{x\to 1} \dfrac{\sqrt{x}^2-\sqrt{1}^2}{x-1} \cdot \dfrac{1}{\sqrt{x}+\sqrt{1}} = \lim_{x\to 1} \dfrac{x-1}{x-1} \cdot \dfrac{1}{\sqrt{x}+\sqrt{1}} \lim_{x\to 1} \dfrac{x-1}{x-1} \cdot \dfrac{1}{\sqrt{x}+\sqrt{1}}$
$= \lim_{x\to 1} \dfrac{1}{\sqrt{x}+\sqrt{1}} = \dfrac{1}{\sqrt{1}+\sqrt{1}} = \dfrac{1}{2}$, where the next to the last equality follows since $\sqrt{x}$ is continuous at 0.

12.

a. Since $s = 90t^{1/2} - 25t^{3/2} + 3t^{5/2}$, $v = 90 \cdot \frac{1}{2}t^{-1/2} - 25 \cdot \frac{3}{2}t^{1/2} + 3 \cdot \frac{5}{2}t^{3/2}$ or $v = \frac{90}{2}t^{-1/2} - \frac{75}{2}t^{1/2} + \frac{15}{2}t^{3/2} = \frac{15}{2}t^{-1/2}\big(6-5t+t^2\big)$. Hence $v(1) = \frac{15}{2} \cdot (6-5+1) = 15$.

b. We must solve $v(t) = 0$ or $0 = 6 - 5t + t^2 = (t-3)(t-2)$ so $t = 2$ and $t = 3$.

c. Since there are no zeros of the velocity function between 0 and 1 the distance travelled is equal to the displacement which is $s(1) - s(0) = \big(90 - 25 + 3\big) - (0) = 68$.

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13. We know $f(x)$ is continuous at any point $x \neq 1$ since there the function is given by a polynomial and polynomials are continuous. By the same reasoning, it is easy to calculate one sided limits at $x = 1$: $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} x^3 + cx = 1 + c$ since $x^3 + cx$ is continuous; $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x = 1$ since x is continuous. For f to be continuous at $x = 1$ we must have $\lim_{x\to 1} f(x) = f(1) = 1$ so $1 + c = 1$ or $c = 0$.

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14. First we write down the equation for the tangent line at a: $y - f(a) = f'(a)(x-a)$ and, since $f(x) = x^2 + 1$, we have $f'(x) = 2x$, and or line is $y - (a^2+1) = (2a)(x-a)$. This line passes through $(0,-3)$ whenever $(-3) - (a^2+1) = (2a)(0-a)$. This yields $-a^2 - 4 = -2a^2$ or $a^2 = 4$ and $a = \pm 2$.

A second approach proceeds as follows. At the point (x, x^2+1) on the curve, the slope of the tangent line is $2x$ and the slope of the line from this point to $(0,-3)$ is $\frac{x^2+1-(-3)}{x-0} = \frac{x^2+4}{x}$. Equating these two slopes yields $2x = \frac{x^2+4}{x}$ which again yields $x = \pm 2$.
