

Name: _____

Instructor: _____

If Hind please indicate 10:40 or 1:55

Final Exam
December 14, 2001

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for two hours.
- You will only hand in this page, so be sure you have marked the answer sheet below correctly. Dotted lines and new columns indicate page breaks in the test.
- Be sure that you have all 14 pages of the test.

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

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|-----|-------|-----|-----|-----|-----|-----|-------|-----|-----|-----|-----|
| 1. | (a) | (b) | (c) | (d) | (e) | 13. | (a) | (b) | (c) | (d) | (e) |
| 2. | (a) | (b) | (c) | (d) | (e) | 14. | (a) | (b) | (c) | (d) | (e) |
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| 12. | (a) | (b) | (c) | (d) | (e) | 24. | (a) | (b) | (c) | (d) | (e) |
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| | | | | | | 25. | (a) | (b) | (c) | (d) | (e) |

Final Exam Total: _____

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Multiple Choice

1.(6 pts.) Find the limit $\lim_{x \rightarrow 1} \frac{\sin(x - 1)}{x - 1}$.

- (a) 0 (b) -1 (c) 1 (d) $+\infty$ (e) Does not exist.

2.(6 pts.) Find the limit $\lim_{x \rightarrow 0} \frac{\sqrt{x + 9} - 3}{x}$.

- (a) -3 (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) 3 (e) Does not exist

3.(6 pts.) If f and g are two continuous functions with $f(1) = 2$ and $\lim_{x \rightarrow 1}[3f(x) + g(x)] = 6$, find $g(1)$.

- (a) 1 (b) 2 (c) -3 (d) 0 (e) 3

4.(6 pts.) For what value of the constant c is the function f continuous on $(-\infty, \infty)$ where $f(x) = cx + 1$ for $x \leq 2$, $f(x) = cx^2 - 1$ for $x > 2$?

- (a) 2 (b) -1 (c) 3 (d) 4 (e) 1

5.(6 pts.) Find an equation for the tangent line to the curve $y = \frac{1}{\sqrt{x}}$ at the point $(1, 1)$.

- (a) $2y = -x - 3$ (b) $2y = -x + 3$ (c) $2y = x - 3$ (d) $2x = -y + 3$
(e) $2y = x + 3$

6.(6 pts.) If $f(3) = 4$, $g(3) = 2$, $f'(3) = -6$ and $g'(3) = 5$, find $\left(\frac{f}{g}\right)'(3)$.

- (a) -1 (b) -8 (c) 8 (d) 0 (e) $\frac{1}{8}$

7.(6 pts.) The solution to the initial value problem $\frac{dy}{dx} = \sec(x^2)$; $y(0) = 1$ is given by

(a) $y = 1 + \int_0^{x^2} \sec(t) dt$ (b) $y = \int^{x^2} 10 \sec(t) dt$ (c) $y = \int_0^x \sec(t^2 + 1) dt$

(d) $y = 1 + \int_0^x \sec(t^2) dt$ (e) $y = \int_1^x \sec(t^2) dt$

8.(6 pts.) The position function of a particle is given by $s = t^3 - 4.5t^2 - 7t$, for $t \geq 0$. When does the particle reach a velocity of 5 ft/sec ?

(a) 7 seconds (b) 0 seconds (c) 4 seconds (d) 5 seconds (e) 2 seconds

9.(6 pts.) If $g(3) = 6$, $g'(3) = 4$, $f'(3) = 2$, $f'(6) = 7$ and $F(x) = f(g(x))$, find $F'(3)$.

- (a) 24 (b) 6 (c) 28 (d) 42 (e) 14

10.(6 pts.) Let $x^2 - y^2 = 1$. Find $\frac{dy}{dx}$.

- (a) $\frac{x}{y}$ (b) $\frac{y}{x}$ (c) $2x - 2y$ (d) $-\frac{y}{x}$ (e) $-\frac{x}{y}$

11.(6 pts.) Find the equation of the tangent line to the curve $y^2 = x^3(2 - x)$ at the point $(1, 1)$.

- (a) $y = x + 1$ (b) $y = \frac{1}{x}$ (c) $y = -x$ (d) $y = x - 1$ (e) $y = x$

12.(6 pts.) Find the linearization $L(x)$ of $f(x) = x^3$ at the point $a = 1$.

- (a) $3x - 2$ (b) $3x + 1$ (c) $2x - 3$ (d) $2x + 3$ (e) $x - 1$

13.(6 pts.) Find the maximum value of $f(x) = \frac{x}{x+2}$ on the closed interval $[3, 4]$.

- (a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{5}$ (e) $\frac{3}{4}$

14.(6 pts.) Find all numbers $c \in [-1, 1]$ that satisfy $f'(c) = \frac{f(1) - f(-1)}{2}$
for $f(x) = 3x^2 + 2x - 5$.

- (a) -5 (b) 3 (c) 1 (d) -1 (e) 0

15.(6 pts.) What are the x -coordinates of all the inflection points of
 $f(x) = x^4 - 6x^2 + 100x + 99$.

- (a) $x = -1$ and $x = 1$ (b) $x = -6$ (c) $x = 0$
(d) $x = 0$ and $x = -1$ (e) $x = -6$ and $x = 1$

16.(6 pts.) Find all the slant asymptotic lines of $y = f(x) = \frac{x^2 + x + 1}{x}$.

- (a) $y = \frac{1}{x}$ (b) $y = x - 1$ (c) $y = x + 1$ (d) $y = -x + 1$ (e) $y = x$

17.(6 pts.) Find **all** the points on the hyperbola $y^2 - x^2 = 4$ that are closest to the point $(2, 0)$.

- (a) $(1, \sqrt{5})$ (b) $(\sqrt{5}, 1)$ (c) $(1, \pm 5)$ (d) $(-1, \sqrt{5})$ (e) $(1, \pm\sqrt{5})$

18.(6 pts.) If $f(x) = \int_0^{5x} \cos(u^2) du$, find $f'(x)$.

- (a) $5 \cos(25x^2)$ (b) $-25 \cos(5x^2)$ (c) $-5 \cos(5x^2)$ (d) $5 \cos(5x^2)$
(e) $-5 \cos(25x^2)$

19.(6 pts.) Evaluate the integral $\int_0^{\sqrt{\pi}} x \sin(x^2) dx$.

- (a) $\frac{1}{4}$ (b) 2 (c) 1 (d) $1 - \frac{1}{\pi}$ (e) $\frac{\pi}{4}$

20.(6 pts.) Compute the area of the region below the curve $y = 2x$ and above the curve $y = x^2 - 4x$.

- (a) $\int_0^6 ((x^2 - 4x) - 2x) dx$ (b) $\int_0^4 (2x - (x^2 - 4x)) dx + \int_4^6 ((x^2 - 4x) - 2x) dx$
(c) $\int_0^4 ((x^2 - 4x) - 2x) dx$ (d) $\int_0^6 (2x - (x^2 - 4x)) dx$
(e) $\int_0^4 (2x - (x^2 - 4x)) dx$

21.(6 pts.) The shape of a football can be obtained by rotating the curve $y = x - x^2$ around x -axis for $0 \leq x \leq 1$. Compute its volume.

- (a) $\pi \int_0^1 (x^2 - x^4) dx$ (b) $2\pi \int_0^1 (x - x^2) dx$ (c) $\pi \int_0^1 ((x-x^2)^2 - 1^2) dx$
(d) $\pi \int_0^1 (x - x^2)^2 dx$ (e) $2\pi \int_0^1 x(x - x^2) dx$

22.(6 pts.) The solid is obtained by rotating about the y -axis the region bounded by $y = 2 + \sin x$, $y = \sin x$, $x = 0$ and $x = 2\pi$. Find its volume.

- (a) $8\pi^2$ (b) $4\pi^3$ (c) π^3 (d) $2\pi^3$ (e) $8\pi^3$

23.(6 pts.) An ice cube is melting at a rate of $3\text{cm}^3/\text{hr}$. Assume it remains a cube during the melting process. How fast is the length of a side changing when this length is 2cm ?

- (a) 3 cm/sec (b) $\frac{1}{4}\text{ cm/sec}$ (c) 2 cm/sec (d) $\frac{3}{8}\text{ cm/sec}$ (e) $\frac{8}{3}\text{ cm/sec}$

24.(6 pts.) If $f(x) = \int_0^x \sec(t) dt$, find $f''(x)$.

- (a) $\sec(x^2)$ (b) $\sec(x)$ (c) 0 (d) $\sec(x) \tan(x)$
(e) $\cot(x)$

25.(6 pts.) Find the average value of the function $f(x) = 4x - x^2$ on the interval $[0, 3]$.

- (a) 4 (b) 3 (c) 9 (d) $\frac{1}{4}$ (e) $\frac{1}{3}$

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