Multiple Choice

1.(5 pts.) Find the derivative of $f(x) = \sin^2(x^2+1)$.

(a) $f'(x) = 2\cos(2x)$

(b) $f'(x) = \cos^2(x^2+1)$

(c) $f'(x) = \cos^2(x^2+1) \cdot 2x$

(d) $f'(x) = 2\sin(x^2+1) \cdot \cos(x^2+1)$

(e) $f'(x) = 2\sin(x^2+1) \cdot \cos(x^2+1) \cdot 2x$

2.(5 pts.) The function $f(x) = \dfrac{-1}{x^2+4}$, defined for all x,

(a) has a local minimum, which is not an absolute one

(b) has a local maximum

(c) has an absolute minimum, but no other extrema (local or absolute)

(d) has an absolute minimum and an absolute maximum

(e) has exactly one critical point, which is neither a maximum nor a minimum.

3.(5 pts.) Mr Smith starts the engine of his car at 7:30am and drives to work. At 7:50am, he leaves his car at the parking lot of his workplace, which is exactly 10 miles away from his home. Note that 20 minutes is $\frac{1}{3}$ of an hour.

You may assume that the distance–time function is differentiable for any car. Which of the following conclusions hold?

(a) The mean value theorem will tell us the acceleration at any time during his trip.

(b) On his trip to work, Mr Smith must have had a speed of 30mi/h at least once.

(c) His instantaneous speed will have been 30mi/h no more than twice.

(d) We cannot know whether he ever had the speed 30mi/h on this trip.

(e) He was 5mi from home at 7:40am.

4.(5 pts.) Suppose the acceleration of an object moving along a coordinate line is given by $a(t) = 3t^2$. If at time $t = 0$, the position of the object is $s(0) = 5$ and the velocity is $v(0) = 1$, what is the position of the object when $t = 2$?

(a) 11 (b) 9 (c) 10 (d) 12 (e) 7

5.(5 pts.) Let

$$f(x) = 3x(x-4)^{\frac{1}{3}}.$$

Then

$$f'(x) = 4(x-3)(x-4)^{-\frac{2}{3}}$$

and

(a) f has a local maximum at $x = 3$ and another critical point $x = 4$, which is not a local extremum point.

(b) $x = 3$ is the only critical point and f has a local minimum at this point.

(c) f has a local minimum at $x = 3$ and another critical point $x = 4$, which is not a local extremum point.

(d) f has a local minimum at $x = 3$ and a local maximum at $x = 4$.

(e) $x = 3$ is the only critical point and f has a local maximum at this point.

6.(5 pts.) Let $y = t^2 - t - 1$ and $x = t^2 + t + 1$. Which formula below is the differential dy when $t = 1$?

(a) $3\,dx$ (b) $2\,dx$ (c) $\frac{1}{2}\,dx$ (d) $\frac{1}{3}\,dx$ (e) dx

7.(5 pts.) Let $f(x) = x^4 - 2x^2$. Then the extreme values of f over the **interval** $[0, 2]$ are:

(a) Absolute maximum value is 8. Absolute minimum value is -1.

(b) Absolute maximum value is 0. Absolute minimum value is -1.

(c) Absolute maximum value is 8. Absolute minimum value is -2.

(d) Absolute maximum value is 16. Absolute minimum value is -1.

(e) No absolute maximum value. Absolute minimum value is -1.

8.(5 pts.) Use the linearization of the function $f(x) = x^{\frac{1}{10}}$ at $x = 1$ to estimate $(1.01)^{\frac{1}{10}}$. One obtains

(a) 0.99 (b) 1.001 (c) 1.01 (d) 0.999 (e) 1.0001

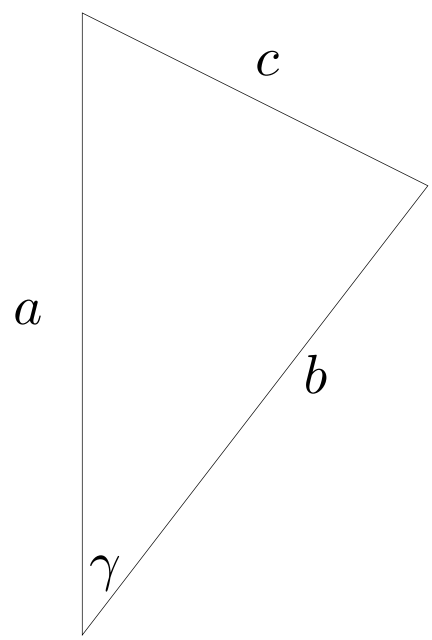
Partial Credit

9.(12 pts.) The point $(3, 2)$ lies on the curve given by the equation

$$y^3 + y + x^3 + x = 40 \ .$$

Calculate the slope of the tangent to this curve at this point.

10.(12 pts.) The hour hand of a large clock at a public building is 4ft long, the minute hand has 6ft. What is the rate of change of the angle between the two hands? At what rate is the distance between the tips of both hands changing, when it is 2 o'clock?
Hints: You may use the *law of cosines*, which relates the quantities depicted in the figure for an arbitrary triangle by the formula $c^2 = a^2 + b^2 - 2ab\cos\gamma$:

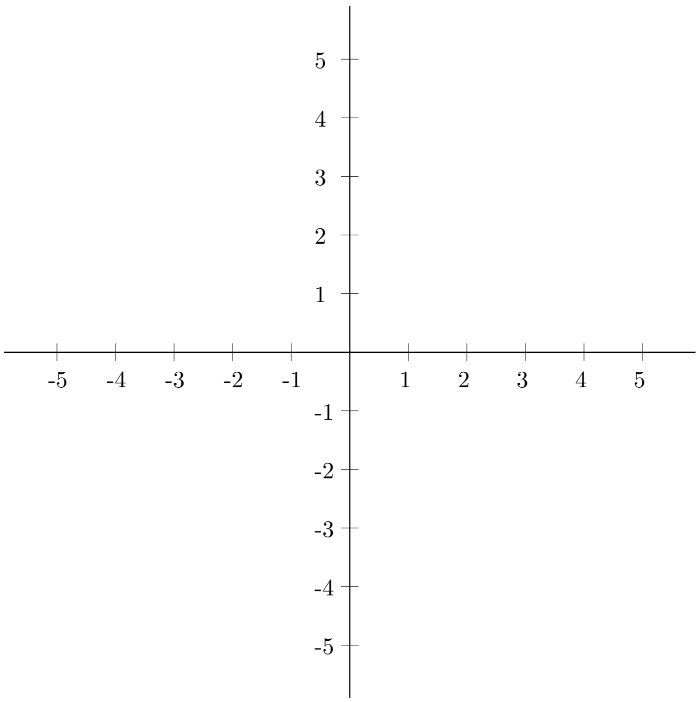


You also know enough to find γ at 2 o'clock but finding $\dfrac{d\gamma}{dt}$ at 2 o'clock is more difficult. It helps to recall that both the hour hand and the minute hand revolve at fixed rates.

11.(12 pts.)

$$f(x) = 1 - 9x - 6x^2 - x^3.$$

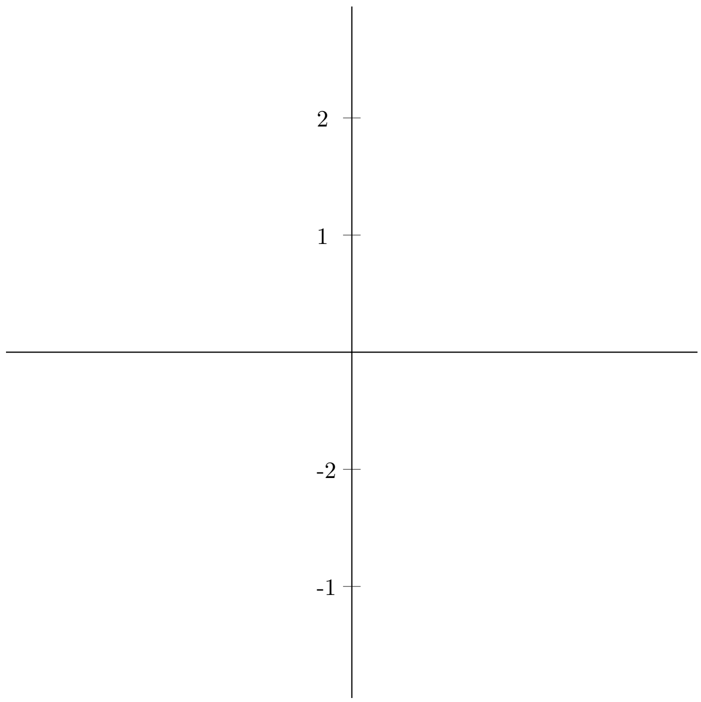
Find intervals where f is (a) increasing (b) decreasing (c) concave up (d) concave down. Identify local extreme values and locate inflection points. Then sketch the graph of f.



12.(12 pts.) For the autonomous differential equation

$$\frac{dy}{dx} = y^2 - y$$

find all equilibrium values and judge which are stable and which are unstable. Construct a phase line and sketch at least one solution curve in each region divided by equilibrium solutions.



13.(12 pts.) $320 are available to fence in a rectangular garden. The fencing for the side of the garden facing the road costs $6 per foot and the fencing for the other three sides costs $2 per foot. Find the dimensions of the garden with the largest possible area.

Name: ANSWERS

Instructor: ANSWERS

Exam II
October 24, 2000

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for one hour.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 10 pages of the test.

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(•)
2.	(a)	(b)	(•)	(d)	(e)
3.	(a)	(•)	(c)	(d)	(e)
4.	(•)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(•)	(d)	(e)
6.	(a)	(b)	(c)	(•)	(e)
7.	(•)	(b)	(c)	(d)	(e)
8.	(a)	(•)	(c)	(d)	(e)

DO NOT WRITE IN THIS BOX!

Total multiple choice:	
9.	
10.	
11.	
12.	
13.	
Total:	