Multiple Choice

1.(5 pts.) Evaluate

$$\int y^2 \cos \frac{y^3}{3}\, dy\ .$$

The answer is:

(a) $\sin \frac{y^3}{3} + C$ (b) $\frac{y^3}{3} \sin \frac{y^3}{3} + C$ (c) $3 \sin \frac{y^3}{3} + C$ (d) $y^2 \sin \frac{y^3}{3} + C$

(e) $y^2 \sin \frac{y^4}{12} + C$

2.(5 pts.) Given the following values, what is the value of $\int_1^2 f(x)\, dx$?

$$\int_1^{10} f(x)\, dx = 10;\ \int_4^{10} f(x)\, dx = 5;\ \int_2^4 f(x)\, dx = 6.$$

(a) 1 (b) 21 (c) -1 (d) 0

(e) The answer can not be determined from the data.

3.(5 pts.) Let $f(x) = \int_0^{1+x^2} \frac{1}{t^3+1}\, dt$. Evaluate $f'(2)$.

(a) $\frac{2}{9}$ (b) $\frac{1}{126}$ (c) $\frac{5}{9}$ (d) $\frac{1}{9}$ (e) $\frac{2}{63}$

4.(5 pts.) The equation $x^4 + x - 1 = 0$ has exactly one positive solution, which is near 1. Which answer below is the result of one iteration of Newton's method applied to this equation with 1 as the starting point?

(a) $\frac{3}{4}$ (b) $\frac{4}{5}$ (c) 1 (d) $\frac{5}{4}$ (e) $\frac{4}{3}$

5.(5 pts.) Which of the following is a correct solution of the initial value problem

$$\frac{dy}{dx} = \frac{1}{3+\sin x}\ , \qquad y(1) = 2$$

(a) $y(x) = 1 + \int_1^x \frac{1}{3+\sin t}\, dt$

(b) $y(x) = 2 + \int_1^x \frac{1}{3+\sin t}\, dt$

(c) $y(x) = 2 + \int_1^{\frac{1}{3+\sin x}} t\, dt$

(d) $y(x) = 1 + \int_2^x \frac{1}{3+\sin t}\, dt$

(e) $y(x) = 1 + \int_1^{x^2} \frac{1}{3+\sin t}\, dt$

6.(5 pts.) Evaluate

$$\sum_{k=1}^{100} k^{-1/2} - \sum_{k=1}^{99} k^{-1/2}$$

(a) $99^{-1/2}$

(b) $100^{-1/2} - 99^{-1/2}$

(c) $\int_1^{100} x^{-1/2}\, dx - \int_1^{99} x^{-1/2}\, dx$

(d) $\frac{1}{10}$

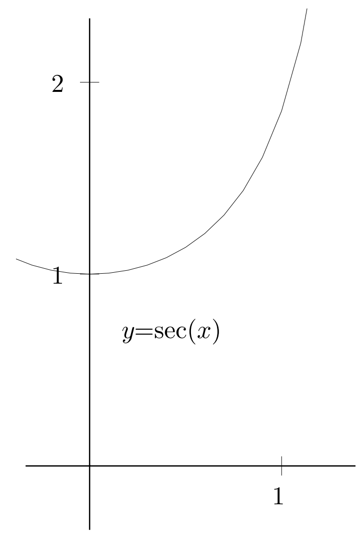
(e) Cannot be determined without knowledge of the value of k.

7.(5 pts.)

$$\int_0^{\pi/2} (1 - \sin^2 x)\, \cos x\, dx = ?$$

(a) $\frac{\pi}{2} - \frac{\pi^3}{24}$ (b) 1 (c) $\frac{\pi}{2} + \frac{\pi^3}{24}$ (d) $\frac{2}{3}$ (e) $\frac{4}{3}$

8.(5 pts.) Let $f(x) = 1/\cos(x) = \sec(x)$ and consider the definite integral $\int_0^1 f(x)\,dx$? Divide the interval of integration into 5 equal pieces. Which sum below is the Riemann sum for this partition where the point in each interval is a point at which $f(x)$ obtains its minimum in that interval.



(a) $\frac{1}{10}\Big(\sec(0) + 2\sec(\tfrac{1}{5}) + 2\sec(\tfrac{2}{5}) + 2\sec(\tfrac{3}{5}) + 2\sec(\tfrac{4}{5}) + \sec(1)\Big)$

(b) $\frac{1}{2}\Big(\sec(0) + \sec(1)\Big)$

(c) $\frac{1}{5}\Big(\sec(0) + \sec(\tfrac{1}{5}) + \sec(\tfrac{2}{5}) + \sec(\tfrac{3}{5}) + \sec(\tfrac{4}{5})\Big)$

(d) $\sec(1)$

(e) $\frac{1}{5}\Big(\sec(\tfrac{1}{5}) + \sec(\tfrac{2}{5}) + \sec(\tfrac{3}{5}) + \sec(\tfrac{4}{5}) + \sec(1)\Big)$

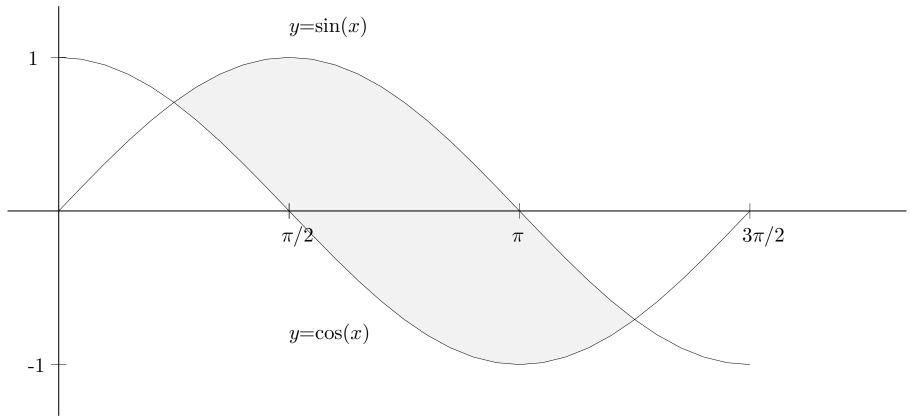
Partial Credit

9.(12 pts.) Write down the formula for Simpson's Rule applied to the integral

$$\int_0^6 \frac{x^2+1}{x^4+1}\,dx$$

where you have divided the interval into 6 pieces.
No credit will be given for simplifying your answer, but points may be deducted for blatant arithmetical errors if you attempt to simplify. (The *style* of the answers for problem 8 is what is wanted here.)

10.(12 pts.) The curves $y = \sin x$ and $y = \cos x$ enclose an area as given in the figure. Set up a definite integral which calculates the area of this region.
No credit will be given for evaluating your integral but points may be deducted for especially "creative" attempts.



11.(12 pts.) Remember that an error estimate for the trapezoidal rule is given by

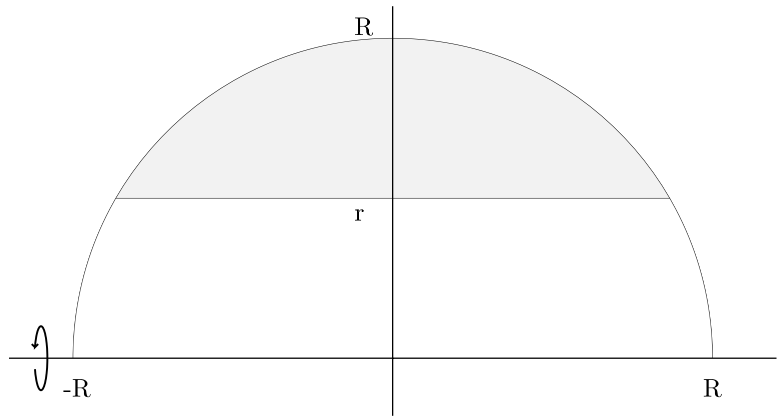
$$(*) \qquad \left| \int_a^b f(x)\,dx - T \right| \le \frac{b-a}{12} h^2 \Big(\max_{x\in[a,b]} \left|f''(x)\right| \Big) \quad ; \quad h = \frac{b-a}{n}$$

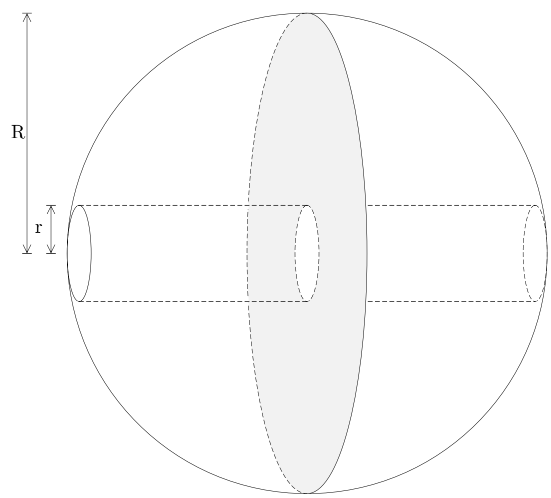
where T is the formula you are expected to know from the Trapezoid Rule and n is the number of subintervals.

Assume you want to approximate $\int_0^2 \sin(x^2)\,dx$ using the trapezoidal rule and will tolerate an error of at most 0.1.

(a) Calculate the second derivative (you should know of which expression) and show that its absolute value is ≤ 18.
(b) Use the formula $(*)$ above with $\max_{x\in[a,b]} \left|f''(x)\right| \le 18$ from (a) to find the smallest number n so that the resulting estimate of the error is less than 0.1.

12.(12 pts.) Out of a solid ball of radius R, a cylindrical hole of radius r ($r < R$) has been drilled centrally (i.e., the axis of the cylinder passes through the center of the ball). Set up a definite integral, in terms of R and r, for the volume of the remaining body.
No credit will be given for evaluating your integral but points may be deducted for especially "creative" attempts.



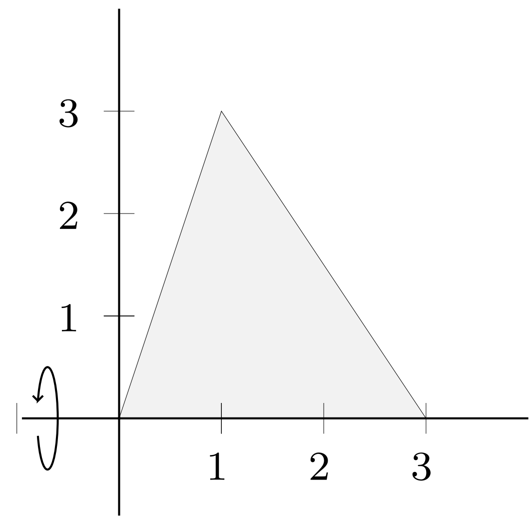


13.(12 pts.) The triangle whose corners have coordinates $(0,0)$, $(1,3)$, $(3,0)$ is rotated around the x-axis to produce a solid of revolution.

(a) Set up a definite integral which gives the volume of this solid.

(b) Evaluate your integral.

Hint: The line $x = \dfrac{y}{3}$ goes through $(0,0)$ and $(1,3)$ while the line $x = -\dfrac{2y}{3} + 3$ goes through $(3,0)$ and $(1,3)$



Name: ANSWERS

Instructor: ANSWERS

Exam III
November 28, 2000

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for one hour.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 10 pages of the test.

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(•)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(•)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(•)
4.	(a)	(•)	(c)	(d)	(e)
5.	(a)	(•)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(•)	(e)
7.	(a)	(b)	(c)	(•)	(e)
8.	(a)	(b)	(•)	(d)	(e)

DO NOT WRITE IN THIS BOX!

Total multiple choice:	
9.	
10.	
11.	
12.	
13.	
Total:	