9. Using $h = \frac{6-0}{6} = 1$, the Simpson's formula gives:

$$S = \frac{1}{3}\Big(\frac{0+1}{1+1} + 4\cdot\frac{1+1}{1+1} + 2\cdot\frac{2^2+1}{2^4+1} + 4\cdot\frac{3^2+1}{3^4+1} + 2\cdot\frac{4^2+1}{4^4+1} + 4\cdot\frac{5^2+1}{5^4+1} + \frac{6^2+1}{6^4+1}\Big)$$

You were explicitly requested not to do any further simplification.

10. To describe the region under consideration, we use x. Since we are doing an area, we need to integrate from the x–coordinate of the left–hand intersection point to the x–coordinate of the right–hand intersection point. The function we need to integrate is the difference of the y–coordinates of the top curve, $y = \sin x$, and the bottom curve $y = \cos x$.

The two curves intersect when $\cos x = \sin x$, or when $\tan x = 1$. This occurs at $x = \frac{\pi}{4}$ and hence *all* the solutions are given by $x = \frac{\pi}{4} + k\pi$, k an integer. This makes the limits of integration $\frac{\pi}{4}$ and $\frac{\pi}{4} + \pi = \frac{5\pi}{4}$ and so the integral is

$$\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x)\, dx\ .$$

11.Part a. The function f under discussion is the function you want to integrate, so $f(x) = \sin(x^2)$. Then, using the Chain Rule,

$$f'(x) = 2x\cos(x^2)$$

and using both the Chain Rule and the Product Rule

$$\begin{aligned} f''(x) &= 2\cos(x^2) + (2x)\big((2x)(-\sin(x^2))\big) \\ &= 2\cos(x^2) - (4x^2)\sin(x^2) \end{aligned}$$

Since both the sin and the cos lie between -1 and $+1$,

$$\big|f''(x)\big| \leq 2 + 4x^2 \ .$$

Since x lies in the interval $[0, 2]$, the biggest $2+4x^2$ can be on this interval is $2+4\cdot 2^2 = 18$.
Part b. We have that

$$\big|E_T\big| \leq \frac{b-a}{12}h^2\Big(\max_{x\in[a,b]}\big|f''(x)\big|\Big) = \frac{(b-a)^3}{12n^2}\Big(\max_{x\in[a,b]}\big|f''(x)\big|\Big)$$

In our case, $b = 2$, $a = 0$ and $\Big(\max_{x\in[a,b]}\big|f''(x)\big|\Big) \leq 18$, so we know

$$\big|E_T\big| \leq \frac{2^3\cdot 18}{12n^2} = \frac{12}{n^2}$$

Hence, we will be sure that $\big|E_T\big| < 0.1$ if we choose n so that $\frac{12}{n^2} < 0.1$. This will be guaranteed provided $12 < (0.1)n^2$ or $120 < n^2$. In our problem, n must be a positive integer and the smallest positive integer with $n^2 > 120$ is $n = 11$ with $n^2 = 121$.
Additional Remarks not asked for by the problem. What this calculation shows is that if we apply the Trapezoid Rule with 11 divisions, then we will be sure that the number we obtain will differ from the true answer by less than 0.1. We could also use 12, 13 or 1,000 divisions, but if we use less than 11 we cannot be sure that we are within 0.1 of the answer. Notice that for $n = 1000$, we are sure that we are within $\frac{12}{10^6} = .000012$ of the correct answer.

Solving the inequality $\frac{12}{n^2} > 0.1$ (or even $\geq$) tells us how many subdivisions we need to take to be sure that we cannot guarantee that the approximation is within 0.1 of the true answer. (It may well be within than bound, we just cannot guarantee it.) In more prosaic terms, this inequality answers no useful question at all.

Solving the equality $\frac{12}{n^2} = 0.1$ may help you deal with the inequality, but $n = \sqrt{120}$ is certainly not an integer and so for sure is not the answer.

12. This can be done by either shells or washers. If we want to use the washer formula we need to integrate along the x–axis. The limits of integration are easy to describe from the graph. They are the x–coordinates of the intersection points of the horizontal line and the semi–circle. The horizontal line is given by $y = r$ so these points are $\pm\sqrt{R^2 - r^2}$. The formula for the volume then becomes

$$V = \pi \int_{-\sqrt{R^2-r^2}}^{\sqrt{R^2-r^2}} \Big((R^2 - x^2) - r \Big) \, dx$$

This integral is not particularly difficult to evaluate, but requiring you to do so would have made the exam much too long.

If we want to use the shell formula we need to integrate up the y–axis. The lines perpendicular to the y–axis begin to enter our plane region when $y = r$ and stop when $y = R$. Hence the formula for the volume of the solid of revolution is

$$V = 2\pi \int_r^R y \cdot \left(\left(\sqrt{R^2 - y^2}\right) - \left(-\sqrt{R^2 - y^2}\right) \right) dy$$

If you really feel a need to simplify, even though requested not to,

$$V = 4\pi \int_r^R y \cdot \sqrt{R^2 - y^2} \, dy$$

This is not a difficult integral to do either.

13. Again the problem succumbs to either shells or washers. We begin with shells this time. (Especially since we are given the boundary curves in the form $x = g(y)$ and we are rotating about the x–axis.) The integral starts at $y = 0$ and goes to $y = 3$ and the formula is

$$V = 2\pi \int_0^3 y \cdot \Big((3 - \frac{2y}{3}) - \frac{y}{3} \Big)\, dy$$

Since we have been asked to do the integral, the first step is to simplify the integrand:

$$V = 2\pi \int_0^3 y \cdot \Big(3 - \frac{2y}{3} - \frac{y}{3}\Big)\, dy = 2\pi \int_0^3 y \cdot \big(3 - y\big)\, dy = 2\pi \int_0^3 3y - y^2\, dy$$

so

$$V = 2\pi \Big(3 \cdot \frac{y^2}{2} - \frac{y^3}{3}\Big)\Big|_0^3 = 2\pi \Big(\frac{27}{2} - \frac{27}{3}\Big) = 2\pi \frac{27}{6} = 9\pi$$

If we wish to use the washer formula, the integral starts at $x = 0$ and goes to $x = 3$, but at $x = 1$ there is a change in the formula for the top curve. We need to solve the two equations for our lines to get y as a function of x: $x = \frac{y}{3}$ yields $y = 3x$ and $x = -\frac{2y}{3} + 3$ yields $y = \frac{3}{2}\big(3 - x\big)$. We have

$$V = \pi \int_0^1 (3x)^2\, dx + \pi \int_1^3 \Big(\frac{3}{2}\big(3 - x\big)\Big)^2\, dx$$

Hence

$$V = \pi \int_0^1 9x^2\, dx + \pi \int_1^3 \frac{9}{4}\big(9 - 6x + x^2\big)\, dx$$

$$V = 3\pi x^3\Big|_0^1 + \frac{9\pi}{4} \cdot \Big(\big(9x - 3x^2 + \frac{x^3}{3}\big)\Big|_1^3 \Big) = 3\pi + \frac{9\pi}{4} \cdot \Big((27 - 27 + 9) - (9 - 3 + \frac{1}{3}) \Big)$$

$$V = 3\pi + \frac{9\pi}{4} \cdot \big(9 - \frac{19}{3}\big) = 3\pi + \frac{9\pi}{4}\Big(\frac{27}{3} - \frac{19}{3}\Big) = 3\pi + \frac{9\pi}{4} \cdot \frac{8}{3} = 3\pi + 6\pi = 9\pi\ .$$