Multiple Choice

1.(6 pts.) Find the limit $\lim_{x\to 2} \frac{x^3+4}{x^2-1}$.

(a) 4 (b) 2 (c) 6 (d) $+\infty$ (e) Does not exist.

2.(6 pts.) A body moves along a straight line with a constant acceleration of $5\,\text{m/sec}^2$. Initially it is moving at a velocity of $11\,\text{m/sec}$ at a distance of $20\,\text{m}$ from the zero position on the line. Which formula below is a formula for the body's position at time t?

(a) $\frac{11}{2}t^2 + 20$ (b) $\frac{5}{2}t^2 + 11t + 20$ (c) $10t^2 + 11t + 5$ (d) $\frac{5}{2}t^2 + 11t$

(e) $\frac{11}{2}t^2 + 5t + 20$

3.(6 pts.) A 13 ft ladder is leaning against the side of a building when its base begins to slide away from the building. By the time the base is 5 ft from the building, the base is moving at a rate of 4 ft/sec. How fast is the top of the ladder sliding down the wall at this moment?

(a) $\frac{3}{5}$ ft/sec (b) 4 ft/sec (c) $\frac{5}{12}$ ft/sec (d) $\frac{5}{3}$ ft/sec (e) $\frac{1}{4}$ ft/sec

4.(6 pts.) Let $f(x) = \int_2^{1+x^2} \frac{1}{t^2-1}\,dt$. Evaluate $f'(3)$.

(a) $\frac{9}{10}$ (b) $\frac{10}{9}$ (c) $\frac{2}{33}$ (d) $\frac{1}{99}$ (e) $\frac{1}{9}$

5.(6 pts.) Find an equation for the tangent line to the curve $y = x^4 - 15x^2 + 30$ at the point $(2, -14)$.

(a) $y = \frac{1}{28}x - 14$ (b) $y = 32x - 78$

(c) $y = x - 16$ (d) $y = (4x^3 - 30x)(x-2) - 14$

(e) $y = -28x + 42$

6.(6 pts.) $\lim_{x\to\infty} \frac{3x^4 - 5x^3 + 1x^2 - 19x + 11}{5x^4 - 6x^3 + 7x^2 - 78x + 199} = ?$

(a) $\frac{3}{199}$ (b) $\frac{3}{5}$ (c) $\frac{11}{5}$ (d) $\frac{1}{7}$ (e) $\frac{11}{199}$

7.(6 pts.) Which equation below is the solution to the initial value problem

$$\frac{dy}{dx} = \frac{1}{3+\sin^2 x} \quad ; \qquad y(1) = 2$$

(a) $2 + \int_1^x \frac{1}{3+\sin^2 w \cos w}\, dw$

(b) $3 + \int_1^x \frac{1}{2+\sin^2 w}\, dw$

(c) $1 + \int_2^x \frac{1}{3+\sin^2 w}\, dw$

(d) $1 + \int_2^x \frac{1}{3+2\sin w \cos w}\, dw$

(e) $2 + \int_1^x \frac{1}{3+\sin^2 w}\, dw$

8.(6 pts.) Evaluate $\int_1^9 \frac{1}{\sqrt{x}(1+2\sqrt{x})^2}\, dx.$

(a) $\frac{4}{21}$ (b) $\frac{1}{4}$ (c) $\frac{1}{7}$ (d) $\frac{8}{9}$

(e) The integral does not exist.

9.(6 pts.) The equation $x^5+x-1=0$ has one solution between 0 and 1. Find the result of one iteration of Newton's Method applied to this equation with 1 as the starting point.

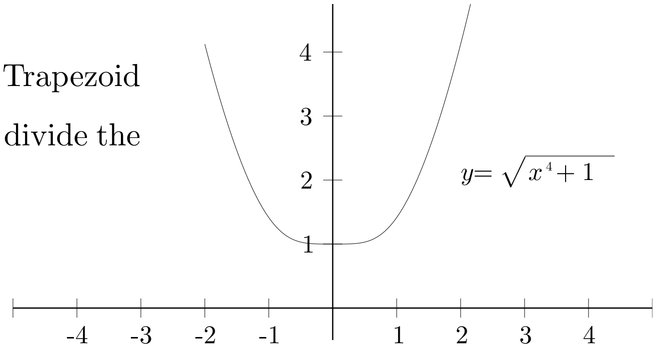
(a) $\frac{3}{4}$ (b) $\frac{5}{7}$ (c) 1 (d) $\frac{5}{6}$ (e) $\frac{1}{2}$

10.(6 pts.) Consider $\int_1^3 x^3\, dx$. Divide the interval of integration into 5 equal pieces. Which summation below is the Riemann sum for this partition where the point in each interval is a point at which $f(x)$ obtains its maximum in that interval?

(a) $\frac{2}{5}\sum_{i=0}^{4}\left(1+\frac{2i}{5}\right)^3$ (b) $\frac{2}{5}\sum_{i=0}^{4}\left(\frac{2i}{5}\right)^3$ (c) $\frac{2}{5}\sum_{i=1}^{5}\left(\frac{2i}{5}\right)^3$

(d) $\frac{1}{5}\sum_{i=1}^{5}\left(1+\frac{2i}{5}\right)^3$ (e) $\frac{2}{5}\sum_{i=1}^{5}\left(1+\frac{2i}{5}\right)^3$

11.(6 pts.)
Which sum below is the result of applying the Trapezoid rule to the integral $\int_{-2}^{2} \sqrt{x^4+1}\, dx$ where we divide the interval into 8 pieces?



(a) $\frac{1}{4}\left(1\cdot\sqrt{17}+2\cdot\frac{\sqrt{97}}{4}-2\cdot\sqrt{2}+2\cdot\frac{17}{4}+2\cdot 1-2\cdot\frac{17}{4}+2\cdot\sqrt{2}-2\cdot\frac{\sqrt{97}}{4}+1\cdot\sqrt{17}\right)$

(b) $\frac{1}{3}\left(1\cdot\sqrt{17}+2\cdot\frac{\sqrt{97}}{4}+4\cdot\sqrt{2}+2\cdot\frac{17}{4}+4\cdot 1+2\cdot\frac{17}{4}+2\cdot\sqrt{2}+4\cdot\frac{\sqrt{97}}{4}+1\cdot\sqrt{17}\right)$

(c) $\frac{1}{4}\left(1\cdot\sqrt{17}+2\cdot\frac{\sqrt{97}}{4}+2\cdot\sqrt{2}+2\cdot\frac{17}{4}+2\cdot 1+2\cdot\frac{17}{4}+2\cdot\sqrt{2}+2\cdot\frac{\sqrt{97}}{4}+1\cdot\sqrt{17}\right)$

(d) $\frac{1}{2}\left(1\cdot\sqrt{17}+2\cdot\frac{\sqrt{97}}{4}+2\cdot\sqrt{2}+2\cdot\frac{17}{4}+2\cdot 1+2\cdot\frac{17}{4}+2\cdot\sqrt{2}+2\cdot\frac{\sqrt{97}}{4}+1\cdot\sqrt{17}\right)$

(e) $\frac{1}{4}\left(1\cdot\sqrt{17}+2\cdot\frac{\sqrt{97}}{4}-4\cdot\sqrt{2}+2\cdot\frac{17}{4}+2\cdot 1-4\cdot\frac{17}{4}+2\cdot\sqrt{2}-4\cdot\frac{\sqrt{97}}{4}+1\cdot\sqrt{17}\right)$

12.(6 pts.) The slope of the tangent line to the curve $y^2 = x^3 - 3x^2 + 2x$ at the point $(3, -\sqrt{6})$ is

(a) $\frac{3}{2\sqrt{6}}$ (b) $\frac{\sqrt{6}}{2}$ (c) $\frac{-\sqrt{6}}{2}$ (d) $-\frac{11}{2\sqrt{6}}$ (e) $\frac{11}{2\sqrt{6}}$

13.(6 pts.) What is $\frac{d^2y}{dx^2}$ for the parameterized curve $x(t) = 1+\sin t$, $y(t) = t+\cos t$ when $t = 0$?

(a) -1 (b) 1 (c) 0 (d) $\tan(1)$

(e) The curve is not differentiable at $t = 0$.

14.(6 pts.) Consider a solid in space which is sliced by planes perpendicular to the x axis. The base of the solid is in the yz plane. At distance $x > 0$ from the yz plane, the slice is a **square** with the length of one side being $\sqrt{1-x^3}$. Which integral below computes the volume?

(a) $\pi \int_0^1 (1-x^3)\, dx$ (b) $2\pi \int_0^1 \sqrt{1-x^3}\, dx$ (c) $2\pi \int_0^1 x\sqrt{1-x^3}\, dx$

(d) $\int_0^1 x\sqrt{1-x^3}\, dx$ (e) $\int_0^1 (1-x^3)\, dx$

15.(6 pts.) Which statement below holds for the autonomous differential equation

$$\frac{dy}{dx} = \frac{y}{1+y^2} \ ?$$

(a) If $y(0) > 0$ then $y(2) > y(0)$. (b) If $y(0) > 0$ then $y(2) < y(0)$.

(c) If $y(0) < 0$ then $y(2) > y(0)$. (d) $y = \frac{2}{5}$ is a solution.

(e) The equation has no solution for which y is a constant.

16.(6 pts.) Consider the region in the first quadrant bounded by the lines $y = 2x + 1$ and $x = 3$. Rotate this region around the y axis. Which integral below computes the volume of the resulting solid of revolution?

(a) $\pi \int_0^3 y(1-y)\, dy$ (b) $2\pi \int_0^3 x(2x+1)\, dx$

(c) $2\pi \int_0^3 3(2x+1)\, dx$ (d) $\pi \int_0^3 (2x+1)^2 - x^2\, dx$

(e) $\pi \int_0^7 \left(3^2 - (2x+1)^2\right) dx$

17.(6 pts.) Where does the graph of the linearization of the function $f(x) = 3x^3 - 12$ at $x = 2$ cross the y axis?

(a) At $y = -12$. (b) At $y = -60$. (c) At $x = 2$. (d) No where. (e) At $y = 24$.

18.(6 pts.) On which interval below is the function $2x^3 - 15x^2 + 24x$ decreasing?

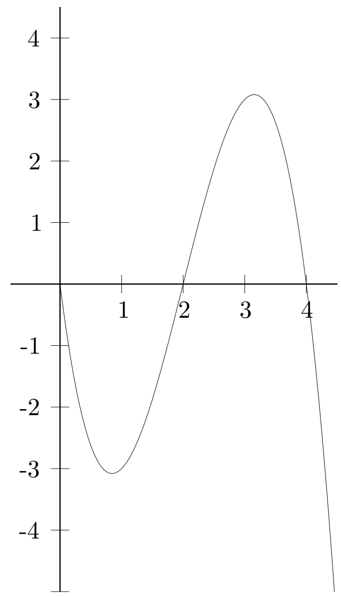
(a) $[0,4]$ (b) $[2,8]$ (c) $[1,4]$ (d) $[3,5]$ (e) $[0,2]$

19.(6 pts.) How many inflection points does the curve $y = 4x^5 - 5x^4 - 9$ have?

(a) None (b) 1 (c) 2 (d) 3 (e) 4

20.(6 pts.)

If the following is a graph of the function $f(x)$ which graph among the answers is the graph of $\int_0^x f(t)\,dt$?



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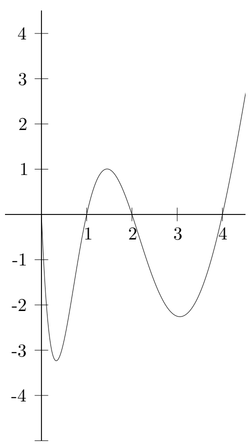

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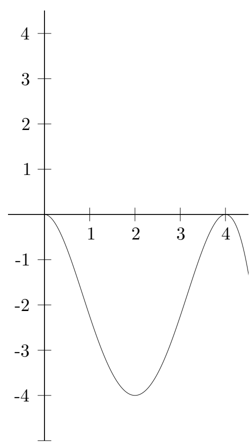

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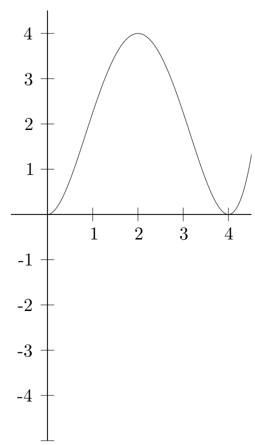

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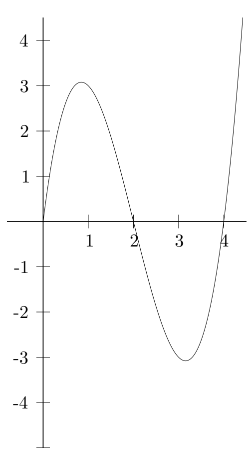

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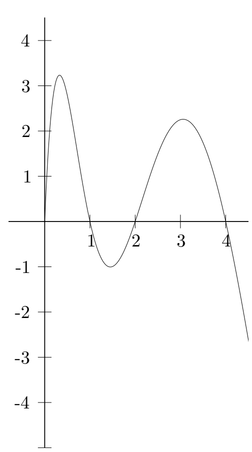


21.(6 pts.) Which answer below identifies **all** of the asymptotes of the curve

$$y = \frac{x^2 + 2x + 1}{x - 1} \ ?$$

(a) $x = 1$ is a vertical one; $y = -1$ is a horizontal asymptote.

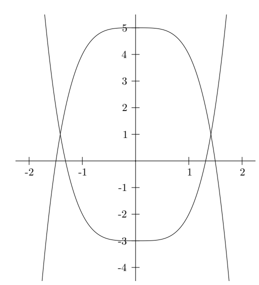
(b) $x = 1$ is a vertical asymptote.

(c) $x = 1$ is a vertical asymptote; $y = 1$ is a horizontal one.

(d) $x = 1$ is a vertical asymptote; $y = x + 3$ is an oblique one.

(e) $y = 3$ is a horizontal asymptote.

22.(6 pts.)
The curves $y = x^4 - 3$ and $y = -x^4 + 5$ enclose an area. Set up a definite integral which calculates the area of this region.



(a) $\int_{-1}^{1} (8 - 2x^4)\, dx$

(b) $\int_{-1}^{1} 2\, dx$

(c) $\int_{-\sqrt{2}}^{\sqrt{2}} (8 - 2x^4)\, dx$

(d) $\int_{0}^{\sqrt[4]{3}} (8 - 2x^4)\, dx$

(e) $\int_{-\sqrt{2}}^{\sqrt{2}} 2\, dx$

23.(6 pts.) Evaluate $\int (1 - \sin^2 x) \cos x\, dx$.

(a) $\frac{1}{3} \sin(3x) - \frac{2}{3} \cos^2 x + C$

(b) $\cos x - \frac{\cos^3 x}{3} + C$

(c) $\frac{1}{3} \cos(3x) - \frac{2}{3} \sin^2 x + C$

(d) $\sin x - \frac{\sin^3 x}{3} + C$

(e) $\frac{1}{3} \cos(3x) - \frac{2}{3} \cos^2 x + C$

24.(6 pts.) Which integral below gives the length of the curve $x(t) = 2\cos t$, $y(t) = 5\sin t$ from $t = 0$ to $t = \dfrac{\pi}{2}$?

(a) $\displaystyle\int_0^{\frac{\pi}{2}} \sqrt{4\sin^2 t + 25\cos^2 t}\, dt$

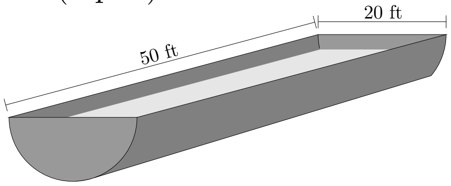
(b) $\displaystyle\int_0^{\frac{\pi}{2}} \sqrt{4\cos^2 t + 25\sin^2 t}\, dt$

(c) $\displaystyle\int_0^{\frac{\pi}{2}} \sqrt{4\sin^2 t + 4\cos^2 t}\, dt$

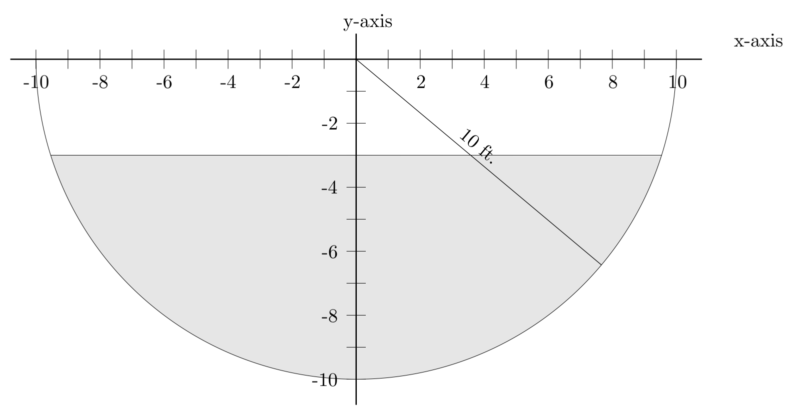
(d) $\displaystyle\int_0^{\frac{\pi}{2}} \sqrt{1 + 4\sin^2 t + 25\cos^2 t}\, dt$

(e) $\displaystyle\int_0^{\frac{\pi}{2}} \sqrt{1 + 4\cos^2 t + 25\sin^2 t}\, dt$

25.(6 pts.)



Find the work done in pumping a liquid over the rim of a tank. The tank is 50 ft long and has a semi–circular end of radius 10 ft. Suppose that the tank is filled to a depth of 7 ft and that the liquid has a density of 100 ft·lbs/ft^3.



(a) $\displaystyle -10^4 \int_{-10}^{-3} y\sqrt{100 - y^2}\; dy$ ft·lbs

(b) $\displaystyle -10^4 \int_{-10}^{-7} y\sqrt{100 - y^2}\; dy$ ft·lbs

(c) $\displaystyle -10^4 \int_{-10}^{-3} \sqrt{100 - y^2}\; dy$ ft·lbs

(d) $\displaystyle -10^4 \int_{-10}^{-7} \sqrt{100 - y^2}\; dy$ ft·lbs

(e) 0 ft·lbs

Name: ANSWERS

Instructor: ANSWERS

Final Exam
December 15, 2000

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for two hours.
- You will only hand in this page, so be sure you have marked the answer sheet below correctly. Dotted lines and new columns indicate page breaks in the test.
- Be sure that you have all 15 pages of the test.

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1. (•) (b) (c) (d) (e)
2. (a) (•) (c) (d) (e)

....

3. (a) (b) (c) (•) (e)
4. (a) (b) (•) (d) (e)

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5. (a) (b) (c) (d) (•)
6. (a) (•) (c) (d) (e)

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8. (•) (b) (c) (d) (e)

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9. (a) (b) (c) (•) (e)
10. (a) (b) (c) (d) (•)

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11. (a) (b) (•) (d) (e)

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12. (a) (b) (c) (•) (e)
13. (•) (b) (c) (d) (e)

14. (a) (b) (c) (d) (•)
15. (•) (b) (c) (d) (e)

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16. (a) (•) (c) (d) (e)
17. (a) (•) (c) (d) (e)

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18. (a) (b) (•) (d) (e)
19. (a) (•) (c) (d) (e)

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20. (a) (•) (c) (d) (e)

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21. (a) (b) (c) (•) (e)
22. (a) (b) (•) (d) (e)

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23. (a) (b) (c) (•) (e)
24. (•) (b) (c) (d) (e)

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25. (•) (b) (c) (d) (e)

Final Exam Total: ____________

Course Total: ____________