11. If we coordinatize so that the ladder is sliding down the y–axis and to the right along the x–axis, then we are being asked to compute $\frac{dy}{dt}$ when $x = 10$ and $\frac{dx}{dt} = 5$.

The relation between x and y is that $x^2 + y^2 = 20^2$. Differentiating with respect to t we get $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$, or $\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$.

When $x = 10$, $10^2 + y^2 = 20^2$ or $y^2 = 300$ or $y = \pm 10\sqrt{3}$ and since we are on the positive side of the y–axis, $y = 10\sqrt{3}$. Hence $\frac{dy}{dt} = -\frac{10}{10\sqrt{3}} \cdot 5 = -\frac{5}{\sqrt{3}}$. We also accepted $\frac{5}{\sqrt{3}}$ as the rate the top of the ladder was sliding *down*.

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12. $L(x) = f(a) + f'(a)(x-a)$ with $f(x) = x^{1/3}$ and $x = 27.3$ and $a = 27$. $f'(x) = \frac{1}{3}x^{-2/3}$, $f(a) = (27)^{1/3} = 3$, $f'(a) = \frac{1}{3}\frac{1}{(27)^{2/3}} = \frac{1}{27}$. Thus, $L(27.3) = 3 + \frac{1}{27}(27.3 - 27) = 3 + \frac{0.3}{27}$.

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13. Differentiate implicitly to get

$$15x^2y' + 30xy + 10 = 2(x^2 + y^2)(2x + 2yy')$$

Set $x = 1$ and $y = 2$ and solve for y'.

$$15y' + 60 + 10 = 2(1 + 4)(2 + 4y')$$

$$50 = 25y'$$

$$y' = 2$$

The tangent line is the line through $(1, 2)$ with slope 2.

$$y = 2 + 2(x - 1)$$

$$y = 2x$$

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14. Since $f(x) = x^3 - 12x$ is continuous on the interval $[-3, 5]$, it must have an absolute minimum and an absolute maximum and these must occur at critical points or at endpoints.

The critical points are given by setting $f'(x) = 3x^2 - 12 = 0$ and solving for x. Since $3x^2 - 12 = 3(x-2)(x+2) = 0$, the critical points are $x = \pm 2$.

Checking the values of $f(x)$ at $x = -3, -2, 2, 5$, we see that $f(-3) = 9$, $f(-2) = 16$, $f(2) = -16$, and $f(5) = 65$.

Thus on the interval $[-3, 5]$, $f(x)$ has an absolute minimum of -16 at $x = 2$ and an absolute maximum of 65 at $x = 5$.

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15. $f(x) = \dfrac{1}{x}$ is differentiable for all real numbers $\neq 0$ since $f'(x) = \dfrac{-1}{x^2}$. Therefore f is continuous for all real numbers $\neq 0$ since a function differentiable at a point is continuous at that point. But then f is continuous on the subinterval $[1,4]$ and differentiable on the subinterval $(1,4)$, so the Mean Value Theorem applies to this function on this interval.

The equation that the MVT asserts has a solution is the equation

$$f'(c) = \frac{f(4) - f(1)}{4 - 1} \quad \text{and} \quad c \in (1,4)$$

For us, $f'(c) = \dfrac{-1}{c^2}$, $f(4) = \dfrac{1}{4}$, $f(1) = 1$, so $\dfrac{-1}{c^2} = \dfrac{\frac{1}{4} - 1}{3} = \dfrac{-\frac{3}{4}}{3} = -\dfrac{1}{4}$, so $c^2 = 4$. This equation has solutions $c = \pm 2$, but the only solution satisfying our additional condition is $c = 2$.
