11. For part a) the definition of the derivative says

$$f'(0) = \lim_{x\to 0} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0} \frac{x\cos\left(\frac{1}{x}\right)-0}{x-0} = \lim_{x\to 0} \cos\left(\frac{1}{x}\right)$$

and you were told that this limit does not exist.

For part b) the definition of the derivative says

$$g'(0) = \lim_{x\to 0} \frac{g(x)-g(0)}{x-0} = \lim_{x\to 0} \frac{x^2\cos\left(\frac{1}{x}\right)-0}{x-0} = \lim_{x\to 0} x\cos\left(\frac{1}{x}\right)$$

and you were told that this limit is 0 so g is differentiable at 0 with $g'(0) = 0$.

12. *Solution 1:* $y' = 2x - 2$ and thus the slope of the tangent line at $x = a$ is $2a - 2$. The equation for the tangent line to the given curve at $(a, a^2 - 2a + 4)$ is $y = (a^2 - 2a + 4) + (2a - 2)(x - a)$. We are to find all such tangent lines that also pass through $(0, 0)$. Setting $y = x = 0$ in the equation for the tangent line, we obtain $0 = (a^2 - 2a + 4) + (2a - 2)(0 - a) = a^2 - 2a + 4 - 2a^2 + 2a = -a^2 + 4$. Thus $a^2 = 4$ and $a = \pm 2$ are the values of x for which the tangent line to the curve at $x = a$ pass through $(0, 0)$. Hence the points are $(2, 4)$ and $(-2, 12)$.

Solution 2: $y' = 2x - 2$ and thus the slope of the tangent line at $x = a$ is $2a - 2$. Using the slope-intercept form of a line with slope $2a - 2$, we have $y = mx + b = (2a - 2)x + b$. If this line is to pass through the origin, it must have y-intercept $b = 0$. Thus $y = (2a - 2)x$. This line must also pass through the point on the curve $(a, a^2 - 2a + 4)$. Substituting this into the equation for the line, we have $a^2 - 2a + 4 = (2a - 2)a$, i.e., $0 = 2a^2 - 2a - a^2 + 2a - 4 = a^2 - 4 = (a - 2)(a + 2)$. Thus $a = \pm 2$ are the values of x for which the tangent line to the curve at $x = a$ pass through $(0, 0)$.

13. The domain of the functions appearing in the equation is all real numbers EXCEPT ZERO (and all of the functions appearing in the equation are continuous on this domain). Therefore, beware of applying the Intermediate Value Theorem (IVT) on a closed interval containing zero.

1. Let $f(x) = \frac{\sin x}{x} - x$. Since $\lim \frac{\sin x}{x} = 1$, it follows that $f(x_0) > 0$ if x_0 is chosen sufficiently small positive. But $f(\pi) = -\pi < 0$. Applying the IVT to f on the interval $[x_0, \pi]$ there is a c such that $f(c) = 0$.

OR, ALTERNATIVELY

2. Let $F(x) = \sin x - x^2$: this function is continuous everywhere. Then $F(\pi) = -\pi^2 < 0$ and $F(\frac{\pi}{6}) = \frac{1}{2} - \frac{\pi^2}{36} > 0$. Hence F vanishes somewhere on $[\frac{\pi}{6}, \pi]$ and so the original equation has a root in this interval.

14. First note that there are three points through which the curve is required to pass, $f(-1) = 3$, $f(0) = 1$ and $f(1) = 0$. The other two conditions determine the slope of the tangent line at the points $(0, 1)$ and $(1, 0)$. Either of the two graphs below are acceptable. The most common errors were drawing graphs that were not continuous or drawing curves that were not the graphs of functions.

$$s'(t) = (12t - t^3)' = 12 - 3t^2 = 0$$

which yields that $t^2 = 4$ and $t = \pm 2$. We choose positive value of $t = 2$. Lastly, find $s(2) = 12 * 2 - 2^3 = 16$ miles.

b) At the moment of impact with the ground $s(t) = 0$. Therefore, we need to solve the following equation for t:

$$s(t) = 12t - t^3 = 0$$

which results in $t = 0, \pm\sqrt{12}$. We choose $t = \sqrt{12}$ and find

$$v(\sqrt{12}) = s'(\sqrt{12}) = 12 - 3 * 12 = -24 \text{ mi./min.}$$

which concludes the solution.