1.(6 pts.) If $f(x) = (x^2+3x)(6x^5-2x^{15})$ compute $f'(1)$.

(a) 30 (b) 40 (c) 10 (d) 36 (e) 20

2.(6 pts.) Compute the left handed limit $\lim_{u\to 1^-} \dfrac{u^2+1}{u^2-1}$

(a) 1 (b) 0 (c) $-\infty$ (d) ∞

(e) Does not exist and is not ∞ or $-\infty$.

3.(6 pts.) Compute the right handed limit $\lim_{y\to -\frac{\pi}{2}^+} \sec y$.

(a) 0 (b) ∞ (c) 1 (d) $-\infty$

(e) Does not exist and is not ∞ or $-\infty$.

4.(6 pts.) If $f(x) = \dfrac{\sqrt{x}+1}{\sqrt[3]{x}}$, then $f'(x) = ?$
Hint: Write f as a sum of two powers of x.

(a) $\frac{1}{6}x^{-\frac{5}{6}} + \frac{1}{3}x^{\frac{4}{3}}$ (b) $\frac{1}{6}x^{-\frac{5}{6}} - \frac{1}{3}x^{-\frac{4}{3}}$ (c) $-\frac{1}{6}x^{-\frac{5}{6}} + 3x^{\frac{4}{3}}$

(d) $\frac{7}{6}x^{\frac{7}{6}} + \frac{3}{2}x^{\frac{4}{3}}$ (e) $6x^{-\frac{5}{6}} - 2x^{-\frac{3}{2}}$

5.(6 pts.) If $f(u) = \sin^2(u^2)$, compute $f'(u)$

(a) $2u\big(\sin^3(u^2)\big)$ (b) $4u\big(\cos^2(u^2)\big)$ (c) $4u\big(\sin(u^2)\big)\big(\cos(u^2)\big)$

(d) $\frac{2}{3}\big(\sin^3(u^2)\big)$ (e) $2u\sin^2(u^2)$

6.(6 pts.) If $y = x\sin y$, find $\dfrac{dy}{dx}$

(a) $\dfrac{\cos y}{1-xy}$ (b) $\dfrac{\sin y}{1-x\cos y}$

(c) $\sin y$ (d) $\sin y + x\cos y$

(e) $x\cos y$

7.(6 pts.) If we define $f(x) = \displaystyle\int_{x^2}^{3} \frac{1}{1+w^2}\,dw$, find $f'(x)$.

(a) $\dfrac{-2x}{1+x^4}$ (b) $\dfrac{-1}{1+x^4}$ (c) $\dfrac{1}{10} - \dfrac{1}{1+x^4}$ (d) $\dfrac{2x}{1+x^2}$ (e) $\dfrac{-2x}{(1+x^2)^2}$

8.(6 pts.) Consider the function $f(x) = x^4 + x^3$ defined on the real line. Which of the following is true.

(a) f has a local minimum at 0

(b) f has a local minimum at both 0 and $-\dfrac{3}{4}$

(c) f has a local minimum at 0 and a local maximum $-\dfrac{3}{4}$

(d) f has no local minima

(e) f has a local minimum at $-\dfrac{3}{4}$

9.(6 pts.) Find the value of x for which the function f below assumes its absolute maximum value on the interval $(0, \infty)$.

$$f(x) = 24 - 2x - \frac{8}{x}$$

(a) 28 (b) 3 (c) 5 (d) 2 (e) 10

10.(6 pts.) On which of the following intervals is the function $f(x) = -\dfrac{x^2}{4} - \cos(x)$ concave upwards?

(a) $\left(\frac{\pi}{6}, \pi\right)$ (b) $\left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$ (c) $\left(-\frac{\pi}{2}, \frac{\pi}{3}\right)$ (d) $\left(-\frac{\pi}{3}, \frac{5\pi}{6}\right)$ (e) $(0, \pi)$

11.(6 pts.) Find an equation of the tangent line to the curve $x^2 + y^2 = 4$ at the point $(1, \sqrt{3})$.

(a) $y = \dfrac{2}{\sqrt{3}}(x - 1) + \sqrt{3}$

(b) $y = -\dfrac{1}{\sqrt{3}}(x - \sqrt{3}) + 1$

(c) $y = \dfrac{1}{\sqrt{2}}(x - 1) + \dfrac{3}{\sqrt{5}}$

(d) $y = -\dfrac{1}{\sqrt{3}}(x - 1) + \sqrt{3}$

(e) $y = \dfrac{2}{\sqrt{3}}(x - \sqrt{3}) + 1$

12.(6 pts.) Find the second derivative of the function $y = \sin(x^2+1) + \cos(x+1)$.

(a) $2x\cos(x^2+1) - 4x\sin(x^2+1) - \cos(x+1)$

(b) $2\cos(x^2+1) - 4x^2\sin(x^2+1) + \cos(x+1)$

(c) $2x\cos(x^2+1) + 4x^2\sin(x^2+1) - \sin(x+1)$

(d) $2\cos(x^2+1) - 4x^2\sin(x^2+1) - \cos(x+1)$

(e) $2\cos(x^2+1) + 4x\sin(x^2+1) + \cos(x+1)$

13.(6 pts.) You have run an experiment which has yielded the following measurements.

t	1.0	1.5	2.0	2.5	3.0
$f(t)$	0.8	1.1	0.9	0.7	0.8

Which number below is the Riemann sum for $\int_1^3 f(t)\,dt$ using **right–hand endpoints** and the given data?
Comment: Answers (a) and (d) are equal. Answer (a) is what you get if you just plug into the right–hand–endpoint–Riemann sum formula; answer (d) is what you get by plugging into the left–hand–endpoint–Riemann sum formula. Since the value of f the same at the beginning and the end, these two answers are equal.

(a) $\frac{1}{2}\big(1.1 + 0.9 + 0.7 + 0.8\big)$

(b) $\frac{1}{2}\big((1.5 \times 1.1) + (2.0 \times 0.9) + (2.5 \times 0.7) + (3.0 \times 0.8)\big)$

(c) $\frac{2}{5}\big(0.8 + 1.1 + 0.9 + 0.7 + 0.8\big)$

(d) $\frac{1}{2}\big(0.8 + 1.1 + 0.9 + 0.7\big)$

(e) $\frac{2}{5}\big((1.0 \times 0.8) + (1.5 \times 1.1) + (2.0 \times 0.9) + (2.5 \times 0.7) + (3.0 \times 0.8)\big)$

14.(6 pts.) Let f be a continuous function and suppose $\int_0^3 f(x)\,dx = 5$. Find $\int_0^9 \frac{f(\sqrt{x})}{\sqrt{x}}\,dx$.

(a) 5/3 (b) 3/2 (c) 5/2 (d) 10 (e) 0

15.(6 pts.) The vertical asymptotes and horizontal asymptotes of the graph of the function

$$f(x) = \frac{\sqrt{x^6+1}}{x(x^2-1)}$$

are:

(a) $x = 0$, $x = 1$, $x = -1$; $y = 1$, $y = -1$ (b) $x = 1$, $x = -1$; $y = 1$

(c) $x = 1$, $x = -1$; $y = 0$, $y = 1$, $y = -1$ (d) $x = 0$, $x = 1$, $x = -1$; $y = 1$

(e) $x = 0$, $x = 1$, $x = -1$; $y = 0$, $y = 1$, $y = -1$

16.(6 pts.) Examine the function

$$f(x) = x^4 - 4x^3 + 4x^2 + 1$$

for regions of increase, decrease, absolute maxima (if any) and absolute minima (if any).
Comment: This problem caused more trouble than any other. The critical points are the roots of $f'(x) = 4x^3 - 12x^2 + 8x = 4x(x-1)(x-2) = 0$: hence $x = 0$, $x = 1$ and $x = 2$. The function f is increasing on the intervals $(0,1)$ and $(2,\infty)$ and decreasing on $(-\infty,0)$ and $(1,2)$. Since $\lim_{x\to\pm\infty} f(x) = \infty$, f has no absolute maxima. There is a local maximum at $x = 1$, with $f(1) = 2$ and two local minima at $x = 0$ and $x = 2$ where $f(x) = 1$. It follows that 1 is an absolute minimum. Hence (c) is the correct answer.

(a) Increasing on $(1,2)$; absolute minimum value of the function is 1

(b) Increasing on $(0,1)$; absolute minimum value of the function is 2

(c) Increasing on $(0,1)$; absolute minimum value of the function is 1

(d) Decreasing on $(1,2)$; absolute minimum value of the function is 2

(e) Decreasing on $(0,1)$; absolute minimum value of the function is 1

17.(6 pts.) Suppose two motorboats leave from the same point at the same time. If one travels north at 3 miles per hour and the other travels east at 4 miles per hour, how fast will the distance between them be changing after 3 hours?

(a) 6 mph (b) 4 mph (c) 7 mph (d) 3 mph (e) 5 mph

18.(6 pts.) A rectangular box with a square base and open top with volume 4000 cm^3 is to be constructed. Find the minimum area of material necessary in its construction.

(a) 800 cm^2 (b) 1400 cm^2 (c) 1200 cm^2

(d) 1600 cm^2 (e) 600 cm^2

19.(6 pts.) A train is traveling at a constant speed of 120 mph (176 ft/sec) when a signal triggers its emergency brakes. The brakes give a constant deceleration of 4 ft/sec^2. How far will the train travel before coming to rest?

(a) 3872 ft. (b) 1760 ft. (c) 5280 ft. (d) 2420 ft. (e) 1936 ft.

20.(6 pts.) Find the area of the region in the right half–plane bounded by the curves $y = x$ and $y = x^2 - 2$.

(a) 8/5 (b) 5/3 (c) 6 (d) 19/3 (e) 10/3

21.(6 pts.) Find the volume of the solid obtained by rotating the region bounded by $y = x$ and $y = \sqrt{x}$ about the line $x = 2$.

(a) 2π (b) $4\pi/3$ (c) $4/15$ (d) $2\pi/15$ (e) $8\pi/15$

22.(6 pts.) The base of a solid is the region in the xy–plane bounded above by $y = \tan x$, below by the x–axis between $x = 0$ and $x = \frac{\pi}{4}$. Slices perpendicular to the x–axis are triangles of height 3. Which integral below is the volume of this solid?

(a) $\frac{3}{2}\int_0^{\frac{\pi}{4}} \tan^2 x \, dx$ (b) $\int_0^{\frac{\pi}{4}} (\pi - \tan x) \, dx$ (c) $\pi \int_0^{\frac{\pi}{4}} \tan^2 x \, dx$

(d) $\frac{3}{2}\int_0^{\frac{\pi}{4}} \tan x \, dx$ (e) $\int_0^{\frac{\pi}{4}} \tan^2 x \, dx$

23.(6 pts.) Which integral below computes the volume of the solid of revolution obtained by rotating the region above the x–axis, below the curve $y = \sin x$ and between $x = 0$ and $x = \pi$, about the line $x = 2\pi$. Use the shell method.

(a) $\pi \int_0^{2\pi} (\pi^2 - \sin^2 x) \, dx$ (b) $2\pi \int_0^{\pi} (\pi^2 - \sin^2 x) \, dx$

(c) $2\pi \int_0^{\pi} (2\pi - x) \sin x \, dx$ (d) $2\pi \int_0^{\pi} x \sin x \, dx$

(e) $2\pi \int_0^{\pi} (\pi - \sin x) dx$

24.(6 pts.) Compute the volume of the solid obtained by rotating the region in the right half–plane bounded between $y = x^2 - 1$ and $y = 1 - x^2$ about the line $x = 1$.

(a) $\frac{5\pi}{3}$ (b) $\frac{8\pi}{7}$ (c) $\frac{10\pi^2}{7}$ (d) $\frac{11\pi}{6}$ (e) $\frac{10\pi}{7}$

25.(6 pts.) Find the average value of the function $y = \cos\big(x/2\big)$ on the interval $[0, \pi]$.

(a) $-\dfrac{1}{\pi}$ (b) 0 (c) $\dfrac{2}{\pi}$ (d) $\dfrac{\pi}{2}$ (e) $\dfrac{2}{3\pi}$

Math 125
Final Exam
December 17, 2003

Name: ______________________

Instructor: ANSWER

- Be sure that you have all 7 pages of the test.
- No calculators are to be used.
- The exam lasts for two hours.
- **When told to begin, remove this answer sheet and keep it under the rest of your test. When told to stop, hand in just this one page.**
- The Honor Code is in effect for this examination, including keeping your answer sheet under cover.

Please mark your answers with an **X**! Do NOT circle them!

The dotted lines in the answer box indicate page breaks.

1.	(a)	(b)	(c)	(d)	(•)	15.	(•)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(•)	(d)	(e)	16.	(a)	(b)	(•)	(d)	(e)
3.	(a)	(•)	(c)	(d)	(e)	17.	(a)	(b)	(c)	(d)	(•)
4.	(a)	(•)	(c)	(d)	(e)	18.	(a)	(b)	(•)	(d)	(e)
5.	(a)	(b)	(•)	(d)	(e)	19.	(•)	(b)	(c)	(d)	(e)
6.	(a)	(•)	(c)	(d)	(e)	20.	(a)	(b)	(c)	(d)	(•)
7.	(•)	(b)	(c)	(d)	(e)	21.	(a)	(b)	(c)	(d)	(•)
8.	(a)	(b)	(c)	(d)	(•)	22.	(a)	(b)	(c)	(•)	(e)
9.	(a)	(b)	(c)	(•)	(e)	23.	(a)	(b)	(•)	(d)	(e)
10.	(a)	(•)	(c)	(d)	(e)	24.	(•)	(b)	(c)	(d)	(e)
11.	(a)	(b)	(c)	(•)	(e)	25.	(a)	(b)	(•)	(d)	(e)
12.	(a)	(b)	(c)	(•)	(e)	Final Exam:	______				
13.	(•)	(b)	(c)	(d)	(e)	Previous Total:	______				
14.	(a)	(b)	(c)	(•)	(e)	Course Total:	______				