11. You may compute the derivative from the definition either of two ways.

1. $$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\frac{1}{x+h+1}-\frac{1}{x+1}}{h} = \lim_{h\to 0}\frac{\frac{(x+1)-(x+h+1)}{(x+1)(x+h+1)}}{h} = \lim_{h\to 0}\frac{\frac{x+1-x-h-1}{(x+1)(x+h+1)}}{h} = \lim_{h\to 0}\frac{\frac{-h}{(x+1)(x+h+1)}}{h} = \lim_{h\to 0}\frac{-1}{(x+1)(x+h+1)} = \frac{-1}{(x+1)(x+1)} = \frac{-1}{(x+1)^2}.$$

OR

2. $$f'(x) = \lim_{y\to x}\frac{f(y)-f(x)}{y-x} = \lim_{y\to x}\frac{\frac{1}{y+1}-\frac{1}{x+1}}{y-x} = \lim_{y\to x}\frac{\frac{(x+1)-(y+1)}{(x+1)(y+1)}}{y-x} = \lim_{y\to x}\frac{\frac{(x-y)}{(x+1)(y+1)}}{y-x} = \lim_{y\to x}\frac{x-y}{(y-x)(x+1)(y+1)} = \lim_{y\to x}\frac{-1}{(x+1)(y+1)} = \frac{-1}{(x+1)^2}.$$

Also acceptable was

$$f'(a) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h} = \lim_{h\to 0}\frac{\frac{1}{a+h+1}-\frac{1}{a+1}}{h} = \lim_{h\to 0}\frac{\frac{(a+1)-(a+h+1)}{(a+1)(a+h+1)}}{h} = \lim_{h\to 0}\frac{\frac{a+1-a-h-1}{(a+1)(a+h+1)}}{h} = \lim_{h\to 0}\frac{\frac{-h}{(a+1)(a+h+1)}}{h} = \lim_{h\to 0}\frac{-1}{(a+1)(a+h+1)} = \frac{-1}{(a+1)(a+1)} = \frac{-1}{(a+1)^2}.$$

For part b), you need to say where $f'(x)$ does not exist. The formula gives the derivative everywhere both the formula and the function are defined. Since there are no roots being taken, the only issue is when are we dividing by 0. The function f itself is undefined when $x+1=0$, or $x=-1$. The derivative is therefore not defined at $x=-1$. The formula for the derivative makes sense everywhere except where $(x+1)^2=0$, or again $x=-1$. Hence the domain of f' is alll real numbers except -1. In interval notation $(-\infty,-1)\cup(-1,\infty)$.

12.

a) $f'(x) = \dfrac{x'(1+x^2)-x(1+x^2)'}{(1+x^2)^2} = \dfrac{1\cdot(1+x^2)-x\cdot 2x}{(1+x^2)^2} = \dfrac{(1-x^2)}{(1+x^2)^2}$.

b) $f'(x)=0$,$x=\,?$

$f'(x) = \dfrac{(1-x^2)}{(1+x^2)^2} = 0$ holds whenever $1-x^2=0$. This last equation has two solutions $x_1=1$ and $x_2=-1$.

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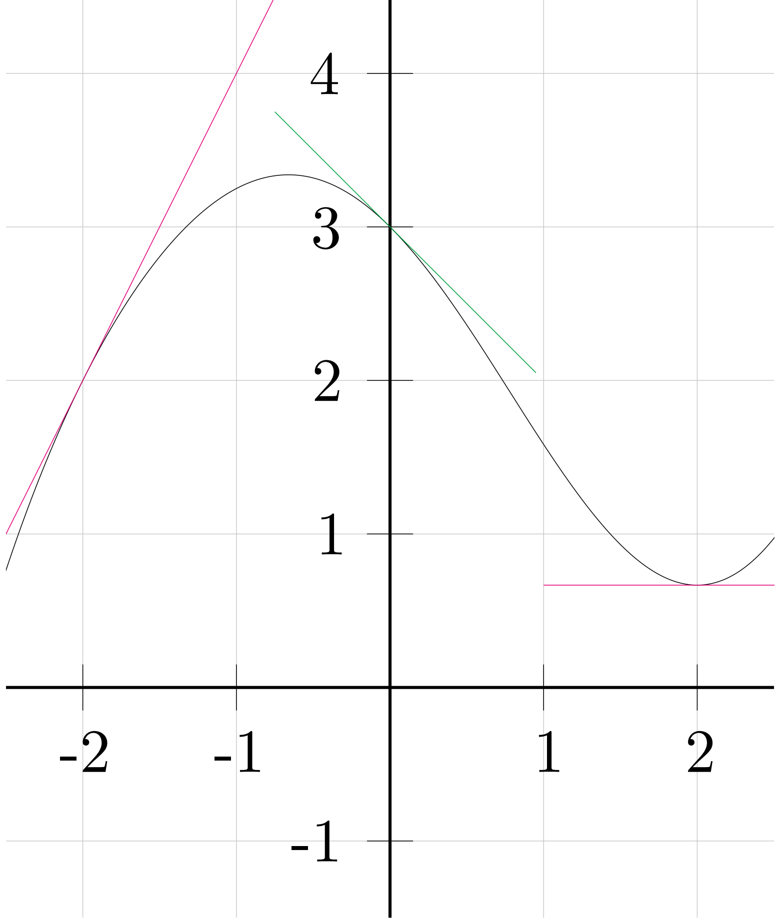
13. $f(x)$ is continuous on $[1,2]$ since it is a rational function and its denominator is not zero on $[1,2]$. By plugging in, $f(1)=-1$ and $f(2)=\dfrac{4}{3}$. Since $f(x)$ is continuous on $[1,2]$ and $f(1)$ is negative and $f(2)$ is positive, it follows from the Intermediate Value Theorem that there must be some c in $(1,2)$ such that $f(c)=0$.

14.



Your graph needs to look something like the graph above. It must go through $(0, 3)$ and have slope -1 there, so the green line must be a tangent line to your graph. The tangent lines to your graph at $x = \pm 2$ must be parallel to the red lines, but your graph may pass through different points, either up or down. Finally, your graph must extend to the edges of the graph in the direction of the x–axis.

15.

(a) Velocity is the instantaneous change in position with respect to time, i.e., the derivative of $s(t)$. Thus $v(t) = s'(t) = \dfrac{d}{dt}(4t - 3t^2 - t^3) = 4 - 3 \cdot 2t^{2-1} - 3t^{3-1} = 4 - 6t - 3t^2$.

(b) When the particle hits the ground, it will be zero meters above the ground, so set $s(t) = 0 = 4t - 3t^2 - t^3 = t(4 - 3t - t^2) = t(4 + t)(1 - t)$. Thus $s(t) = 0$ when $t = -4, 0$ or 1. The root $t = -4$ is not physically valid since time is moving forward; the root $t = 0$ corresponds to the initial time when the ball is tossed into the air; and thus the remaining root $t = 1$ must be when the ball hits the ground after being thrown. Therefore the velocity of the ball when it hits the ground is $v(1) = 4 - 6(1) - 3(1)^2 = -5$ m/s. (Note that the velocity is negative indicating that the ball is traveling towards the surface of planet X when it hits the ground.)
